SCHOOL OF CIVIL ENGINEERING



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HIGHWAY BRIDGE VIBRATION STUDIES

J.T. Gaunt C.D. Sutton





PURDUE UNIVERSITY INDIANA STATE HIGHWAY COMMISSION



Final Report

HIGHWAY BRIDGE VIBRATION STUDIES

TO: H. L. Michael, Director

July 1, 1981

Joint Highway Research Project

Project: C-36-56S

FROM: J. T. Gaunt, Research Engineer

Joint Highway Research Project

File: 7-4-19

The attached Final Report "Highway Bridge Vibration Studies" is submitted as a summary of the extensive research conducted on this project. The research has provided considerable new information and clearer understanding of the bridge vibration problem and advanced capability toward the use of a dynamic-based design criterion for highway bridges. Such use should provide more effectively for the comfort of users of bridges in an economic and beneficial manner.

This research in our opinion has been very productive and we are hopeful the State Highway Department can utilize some of the findings in designing and building a highway bridge and that further confirmation of the findings of this project can be the subject of a new research project on such implementation.

The acceptance of this report as fulfilling the objectives of the Study will terminate this research.

Sincerely,

J. J. Haunt /ms

Research Engineer

JTG:ms

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Final Report

HIGHWAY BRIDGE VIBRATION STUDIES

bу

John T. Gaunt Research Engineer

and

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Joint Highway Research Project

Project No.: C-36-56S

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Conducted by

Joint Highway Research Project Engineering Experiment Station Purdue University

In cooperation with the Indiana State Highway Commission

and the

U.S. Department of Transportation Federal Highway Administration

The contents of this report reflect the views of the author who is responsible for the facts and the accuracy of the data presented herein. The contents do not necessarily reflect the official views or policies of the Federal Highway Administration. This report does not constitute a standard, specification, or regulation.

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The general objectives of this research have been to obtain a better understanding of the dynamic performance of highway bridges and of the vibrations sensed by bridge users in order to aid in the development and implementation of a dynamic-based design criterion which can more effectively ensure the comfort of pedestrians, maintenance workers, cyclists, etc. Because the human body is primarily sensitive to changes in motion, the investigations have focused on accelerations.

Special purpose computer programs were used for parametric studies of the bridge vehicle system. Significant parameters were found to be span length, weight and speed of the vehicle, and the roughness of the bridge deck. Maximum accelerations were only moderately increased by reductions in girder stiffness.

Dynamic responses of some 62 steel and concrete beam bridges under actual highway traffic were measured in the field. Bridges were instrumented with midspan accelerometers and one deflection transducer, all mounted on the curbes of the deck. Differentiated deflection records were used to check corresponding acceleration records. More than 900 crossing records were analyzed for maximum deflection, velocity, acceleration, jerk, and frequency content. Only 5 of the crossing records contained accelerations which exceeded the comfort limit proposed by Wright and Walker.

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HIGHWAY BRIDGE VIBRATION STUDIES

HIGHLIGHT SUMMARY

The general objectives of this research program have been to obtain a better understanding of the dynamic performance of highway bridges and of the vibrations sensed by bridge users in order to aid in the development and utilization of a dynamic-based design criterion which could more effectively ensure the comfort of the users. Specific tasks have included:

- identification through analytical studies of the parameters of the bridge-vehicle system which are most significant in their effect upon the dynamic response of the bridge,
- measurement and analysis of the dynamic performance of common types of highway bridges under actual traffic in the field,
- 3) comparison of analytical predictions with field measurements,
- identification of reasonable dynamic criteria for user sensitivity to vibrations, and
- 5) investigation of a proposed dynamic-based design criterion for controlling bridge vibrations.

This final report is primarily a summary of the research described in greater detail in the three interim reports by Aramraks,

Kropp, and Shahabadi. The first phase of the work, carried out by

Aramraks, consisted of an analytical investigation of the significance of various parameters of the bridge-vehicle system. Special purpose computer programs developed earlier by researchers at the University of Illinois were used for the analysis. Simple span bridges were modeled as plates continuous over flexible beams, while symmetric two and three span bridges were modeled as single continuous beams with



lumped masses. For the simple span program a single axle, two wheel sprung mass was used to represent the vehicle. The multispan bridge program could represent a vehicle as a two or three axle mass with a suspension system.

The human body is primarily sensitive to derivatives of displacements rather than to displacements themselves. Therefore, the analytical investigations focused on accelerations. Significant parameters were found to be span length, weight and speed of the vehicle, and roughness of the bridge deck. Maximum accelerations were only moderately increased by reducing in girder stiffness.

The second phase of the research consisted of an extensive field study of the dynamic responses of some 62 beam-type steel and concrete bridges located throughout the state of Indiana. Data collection was carried out by personnel from the Indiana State Highway Commission Research and Training Center as previously reported by Kropp. Bridges were instrumented with accelerometers mounted on the curbs of the deck near the middle of each span and by a taut wire cantilever beam deflection transducer attached at the location of one of the accelerometers on the traffic lane side of the bridge. Continuous accelerations and deflections produced by actual traffic and by a control vehicle were recorded in analog form on magnetic tape.

The processing and analysis of some 900 single vehicle crossing records were carried out and reported by Kropp. The analog records had to be coverted to digital form for analysis and plotting. The analysis consisted of finding the maximum deflection, velocity, acceleration, and jerk for each vehicle crossing. The frequency content of each record was also determined for comparison with predicted natural frequencies of the bridges. Results indicated that sometimes



the fundamental torsional mode contributed as much to the total bridge motion as did the fundamental bending mode. Algorithms were developed to integrate and differentiate digitized acceleration and displacement records. Excellent comparisons of twice differentiated deflection records with corresponding acceleration records were obtained.

The third phase of the research, carried out by Shahabadi, included a literature survey of human sensitivity to vibrations. Most sensitivity studies consisted of having people who were subjected to steady state harmonic vibrations of various amplitudes and frequencies rate the severity of the vibrations according to some arbitrary descriptive scale ("perceptible," "unpleasant," or "intolerable," for example). Although results are difficult to compare, human sensitivity seems to be most nearly related to the magnitude of the acceleration in the frequency range of concern for highway bridges. Significantly, only a very small fraction of the vehicle crossings monitored in the field produced accelerations which exceeded the limit proposed by Wright and Walker in their important A.I.S.I. report on "Criteria for the Deflection of Steel Bridges."

Because of the importance of roadway roughness shown in the initial analytical studies, the actual profiles of three of the test bridges were carefully measured and used as input data for the comparison of analytical predictions and measured deflections. A method for simulation of roadway roughness was also developed for use with the dynamic response computer programs.

AASHTO Specifications have traditionally attempted to control bridge vibrations by limiting span-depth ratios and live load deflections; however human sensitivity is related to the derivatives of the displacements rather than to the displacements themselves. The 1977



AASHTO specifications were revised to allow the designer the discretion of exceeding these depth-span and deflection limits. Wright and Walker's report is referred to for guidance. Their method has been used to estimate the maximum accelerations of several of the test bridges for comparison with peak accelerations measured in the field under actual traffic.



INTRODUCTION

The goal of the bridge engineer is to design economical structures which are safe, durable, and serviceable. Determination of the dynamic response of bridges has been the topic of numerous studies in recent years. Much of the attention has been focused on maximum dynamic displacements and moments and on the distribution loads to the floor system - information necessary to design for adequate strength. Problems associated with the rapid deterioration of bridge decks have also attracted the interest of researchers.

Another important concern, the comfort of those crossing the bridges, has received relatively little attention. However, transportation agencies do receive occasional comments and complaints from bridge maintenance workers, pedestrians, and passengers in halted vehicles concerning the vibration of bridges. Although humans are subjected to the vibrations of many structures, there is seldom any direct provision in design codes to ensure user comfort. The current AASHTO Specifications (1) impose restrictions upon girder depth-span ratios and upon static deflection - span ratios in the hope that these limits will provide satisfactory dynamic performance. The human body, however, is primarily sensitive to accelerations rather than to displacements. Thus the code requirements may not achieve the desired results.

Unfortunately, the economical use of modern high strength steels
has been hindered somewhat by present code requirements. In order to
meet the deflection and/or depth-span limits, an increase in the quantity



of steel may be required, and the increment of user comfort gained is of questionable significance. Perhaps a satisfactory level of comfort can be provided by more flexible bridges.

The general objectives of this research program have been to obtain a better understanding of the dynamic performance of highway bridges and of the vibrations sensed by bridge users in order to develop a dynamic-based design criterion which would more effectively ensure the comfort of the users. Specific tasks have included:

- identification through analytical studies of the parameters
 of the bridge-vehicle system which are most significant in
 their effect upon the dynamic response of the bridge,
- measurement and analysis of the dynamic performance of typical bridges under actual traffic in the field,
- 3) comparison of field measurements with analytical predictions,
- identification of reasonable quantitative dynamic criteria for user sensitivity to vibrations, and
- 5) investigation of a simple dynamic-based design criterion for controlling bridge vibrations.

The first phase of the research, reported by Aramraks (2), consisted primarily of analytical studies of the effects of varying some of the parameters of the structure-vehicle system. Somewhat surprisingly, the most significant effect was found to be the roughness of roadway. Other important parameters included girder stiffness and vehicle speed.

To expedite the analytical studies existing computer programs were used whenever possible. Two alternatives were strongly considered:



(1) a finite element program which had recently been developed by investigators at the University of Illinois and (2) two somewhat less sophisticated special purpose programs for simply supported and continuous beam bridges developed somewhat earlier at the University of Illinois. Their cooperation in supplying these programs is gratefully acknowledged.

In view of the numerous parametric studies planned, the costs of using the sophisticated but time-consuming finite element program were prohibitive. Thus it was decided to make use of a simple span analysis and program developed by Oran (3) and a multi-span bridge beam program developed by Veletsos and Huang (4). The validity of their analyses had been verified by comparisons with the results of laboratory studies on simply supported beams, as well as with the results of the AASHO Road Test bridges. The analytical models, the methods of analysis, and the results of Aramkaks' study are summarized in Chapter 1.

The second phase of the research, reported by Kropp (5), consisted of an extensive field study of the dynamic responses of 62 beam-type bridges located throughout the state of Indiana. The bridges were instrumented with accelerometers mounted on the curbs of the bridge decks at midspan and by a taut wire cantilever beam deflection transducer attached at the location of the accelerometer on the traffic lane side of the bridge. Continuous accelerations and dynamic deflections produced by a control vehicle and by actual truck traffic were recorded in analog form on magnetic tape. The records were later digitized for plotting and analysis. The analysis consisted of finding the maximum deflection, velocity, acceleration, and jerk for each vehicle



crossing. The frequency content of each record was also determined for comparison with the predicted natural frequencies of the bridges.

Numerical differentiation of the deflection records yielded excellent correlation with the corresponding accelerometer measurements.

The third phase of the research, reported by Shahabadi (6), includes a literature survey of human sensitivity to vibrations. For most of the test programs, subjects were asked to rate according to some arbitrary scale, the severity of steady state harmonic vibrations of various amplitudes and frequencies. Although results are difficult to compare, human sensitivity seems to be most nearly related to the magnitude of the acceleration in the frequency range of concern for highway bridges.

Because of the importance of roadway roughness shown by Aramraks, the actual profiles of three of the test bridges were measured and used as input data for the comparison of analytical predictions and measured accelerations. A method for simulation of roadway roughness was also developed for use with the dynamic response computer program.

Finally, a simple expression has been used to estimate the maximum acceleration of 14 test bridges for comparison with field results.



CHAPTER 1 - ANALYTICAL STUDIES

1.1 METHOD OF ANALYSIS FOR SIMPLE SPAN BRIDGES

The method of analysis for the dynamic response of simple span multi-girder highway bridges used in this project was developed by Oran and Veletsos (3). Their computer program has been modified somewhat for this study to provide more acceleration information and to increase the capabilities for both input and output.

In this analysis the bridge is represented as a plate continuous over flexible beams. Both bending and torsional stiffnesses of the beams are considered, but composite beam-slab action is not. The major steps of the analysis are (1) determination of the instantaneous values of the interacting forces between the vehicle and the bridge and the inertia forces of the bridge itself and (2) evaluation of the deflections and moments produced by these forces.

The second step, which is a problem of statics alone, is solved by a combination of energy principles and the Levy method of analysis for simply supported rectangular plates. Vertical deflections are represented in the form of a double Fourier sine series. The solution has been shown to converge fairly rapidly so that only a limited number of terms is required.

Strain and potential energy of the system are written in terms of the deflections. Enforcing the condition that the total energy must be a minimum yields a set of equations for determining the Fourier coefficients, which can then be substituted into the appropriate moment and deflection expressions.



For the dynamic analysis the mass of the slab is assumed to be uniformly distributed, and the mass per unit length of each beam is assumed to be constant. The vehicle is represented by a single axle, two wheel loading consisting of a sprung mass and two equal unsprung masses. The springs are assumed to be linear elastic and to have equal stiffnesses. Damping has been neglected for both the vehicle and the bridge. The vehicle is assumed to move at constant velocity. The dynamic deflection configuration of the structure is represented by a Fourier series with time dependent coefficients.

Again the total energy of the system can be written in terms of the displacement coordinates and their derivatives. Dead load deflections and roadway surface unevenness are included in the appropriate energy terms. The equations of motion are formulated by application of Lagrange's equation.

The procedures used to evaluate the dynamic response of the bridgevehicle system may be summarized briefly as follows:

- (1) The governing differential equations of motion are solved by means of a step-by-step method of numerical integration to determine the generalized coordinates and their first two derivatives. The time required for the vehicle to cross the span is divided into a number of small intervals, and the governing equations are "satisfied" only at the ends of these intervals by means of an iteration scheme.
- (2) The interacting forces between the vehicle and the bridge and the inerita forces of the bridge are evaluated.
- (3) The dynamic deflections and bending moments induced in the bridge are determined from the dynamic forces acting on the bridge.



1.2 ACCELERATION STUDIES FOR SIMPLE SPAN HIGHWAY BRIDGES

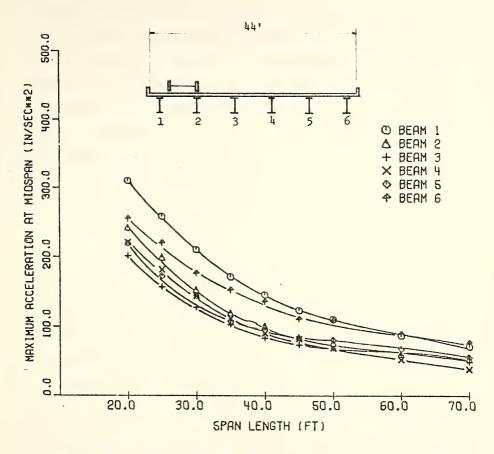
Because of the strong relationship between acceleration and vibration perception, this investigation focuses on the variation of maximum bridge accelerations with certain significant parameters of the bridge-vehicle system. The study is based on the method of analysis and computer program previously described. Bridge data was taken from the Standard Plans of Highway Bridge Superstructures of the U.S. Bureau of Public Roads (7). The study is restricted to steel I-beam bridges with non-composite reinforced concrete decks. The effect of side curbs is not taken into account.

Parameters which affect bridge accelerations can be classified into the following groups: (a) bridge parameters, such as beam span and stiffness, (b) vehicle parameters, such as velocity and transverse position of the wheels; and (c) construction parameters, such as roadway roughness. The accuracy of the analysis is, of course, dependent on such solution parameters as the number of terms chosen for the deflection series expressions and the number of integration steps. For the simple span bridge study, satisfactory convergence was obtained by dividing the time required for the vehicle to cross the span into 400 integration steps. The total computer time required to obtain the response of a 60 ft. span bridge is approximately 150 seconds on a CDC 6500.

1.2.1 Bridge Parameters

The effect of span length upon maximum midspan accelerations is shown in Fig. 1.1. The bridges are standard designs for HS20-44 loading with a 44 ft. roadway width. Six steel beams support the 7-1/2 in.





44 FT ROADWAY HS20-44 LOADING VELOCITY 60 MPH LEVEL SURFACE TWO WHEELS 6 FT WHEEL SPACING RIGHT WHEEL OVER SECOND BEAM

Figure 1.1 Maximum Accelerations at Midspan of B.P.R. Bridges (1 in. = .0254 m, 1 ft. = .305 m)



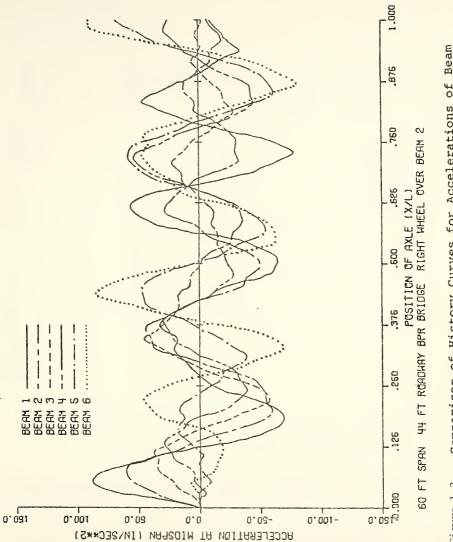
reinforced concrete deck. Beam sizes range from W21X62 for the 20 ft. span to W36X300 for the 70 ft. span. Corresponding fundamental bending frequencies are 16 and 4 Hz.

The vehicle is represented by a single axle two wheel 72 kip loading which moves across the span at a constant 60 mph velocity. The stiffness of each tire spring is 6 kip/in. Unsprung loads and damping are neglected. The vehicle travels along near the edge of the bridge with the inside wheel over beam 2. The exterior beams are shown to have somewhat higher midspan accelerations than the interior beams. Fig. 1.1 also shows a definite increase in maximum accelerations for the standard bridge designs as the span is decreased.

Fig. 1.2 shows the variation of the midspan acceleration of each beam as the vehicle crosses a 60 ft. span. It can be seen that accelerations of beam 1 and beam 6 are out of phase all the time. It is also interesting to note that the maximum acceleration of beam 1 occurs when the vehicle enters the span but the maximum acceleration of beam 6 occurs when the vehicle is at midspan or leaving the bridge.

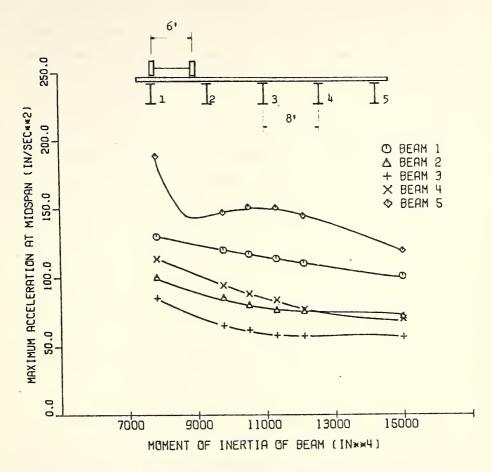
Current bridge specifications attempt to control vibrations by limiting the maximum live load deflection, which may unjustly penalize designs using more flexible high strength steel beams. Figure 1.3 shows the variation of maximum midspan accelerations as beam stiffness is reduced. The basic design utilizes five W36X230 A36 steel beams. Equal strength can be provided by five W36X182 A572-50 beams. Although the maximum acceleration does increase somewhat as stiffness is reduced, the relationship is certainly less severe than an inverse proportion.





Comparison of History Curves for Accelerations of Beam (1 in. = .0254 m)Figure 1.2





5 GIRDER BRIDGE 60 FT SPAN HS20-44 LOADING LEVEL SURFACE TWO WHEELS LEFT WHEEL OVER BEAM 1 VELOCITY 60 MPH

Figure 1.3 Effect of EI of Beam on Maximum Acceleration (1 in. = .0254 m, 1 ft. = .305 m)



1.2.2 Vehicle Parameters

It is well known that vehicle speed has a strong influence on bridge vibrations. In Figure 1.4 maximum midspan accelerations for a 5 beam bridge are plotted as a function of vehicle speed. Over the indicated speed range bridge accelerations are almost directly proportional to vehicle speed.

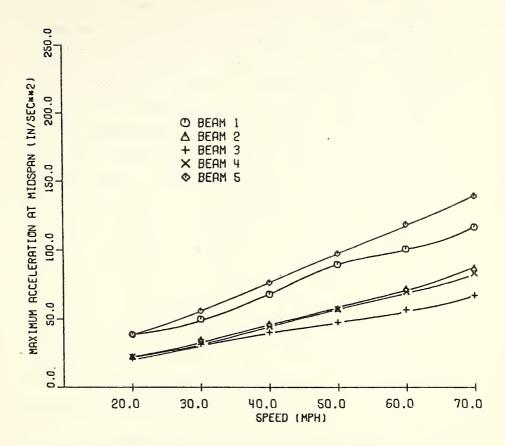
The effect of the transverse position of the vehicle on a 5 beam, 60 ft. span bridge was studied by determining the maximum midspan accelerations for six cases. Results indicate that the accelerations of the edge beams are the greatest when the vehicle travels along the edge of the roadway and tend to decrease when the vehicle travels near the center line of the bridge. In contrast, center beam accelerations increase as the vehicle moves towards the center line and are slightly larger than the edge beam accelerations when the vehicle straddles the center line. For most practical situations, however, edge beam accelerations are larger than those of the interior beams.

1.2.3 Effect of Surface Roughness

Several previous test reports have indicated that surface roughness is a very significant factor affecting the vibration of highway bridges. These reports have recommended that the bridge surface should be as smooth as possible. For all of the previous results reported in this paper, the bridge surface was assumed to be smooth.

The surface roughness is assumed to be represented by some number of half sine waves passing through the supports. Both the number of half sine waves and the amplitude can be varied. It is assumed that





5 GIRDER BRIDGE 60 FT SPAN HS20-44 LOADING LEVEL SURFACE 36WF230 GIRDER TWO WHEELS LEFT WHEEL OVER EDGE BEAM

Figure 1.4 Effect of Speed of Vehicle on Acceleration of Beam (1 in. = .0254 m, 1 ft. = .305 m, 1 mile = 1.61 km)



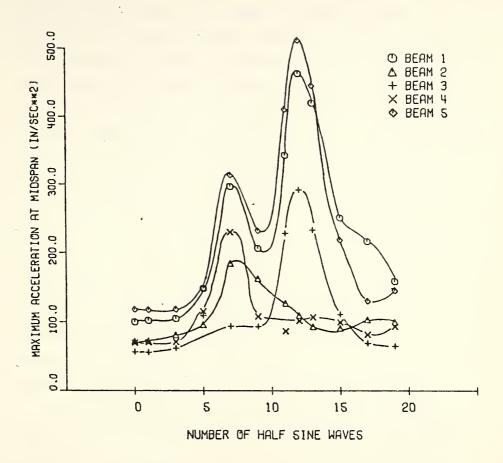
the shape and amplitude of the bridge surface roughness is the same under both wheels of the vehicle.

Figure 1.5 shows the results of determining the maximum midspan accelerations for 0 to 19 half sine waves of 0.5 in. amplitude roadway roughness. Again a 5 girder, 60 ft. span bridge with a 72 kip vehicle traveling 60 mph near the edge of the roadway is used. It is evident that the accelerations are not influenced by up to three half sine waves of roughness; however the effect increases markedly when the number of half waves exceeds five and reaches its peak when the surface roughness consists of twelve half sine waves. For the two peaks at seven and twelve half sine waves, the times required for the vehicle to cross one roughness wave correspond fairly closely to the first two natural frequencies of the bridge. The maximum accelerations with a rough roadway surface are as much as five times those for the same bridge with a smooth deck. Of, course, the probability of a perfectly periodic deck roughness is rather small for a real bridge, but the significance of the effect is certainly obvious. For the nearly resonant condition, maximum accelerations are approximately proportional to the amplitude of the surface roughness.

1.3 METHOD OF ANALYSIS FOR CONTINUOUS BEAM BRIDGES

A general theory for the analysis of continuous bridges was developed by Huang and Veletsos (4). The computer program utilized for this study, which was developed by Huang (8), was previously used by Nieto-Ramirez and Veletsos (9) for an extensive study of the dynamic response of three-span bridges. Application of the program is limited to two and three span symmetric beam bridges.





S GIRDER BRIDGE 60 FT SPAN 36WF230 HS20-44 LØADING 60 MPH LEFT WHEEL OVER BEAM 1 0.5 INCH AMPLITUDE OF ROUGHNESS

Figure 1.5 Effect of Roadway Unevenness on Acceleration (1 in. = .0254 m, 1 ft. = .305 m)

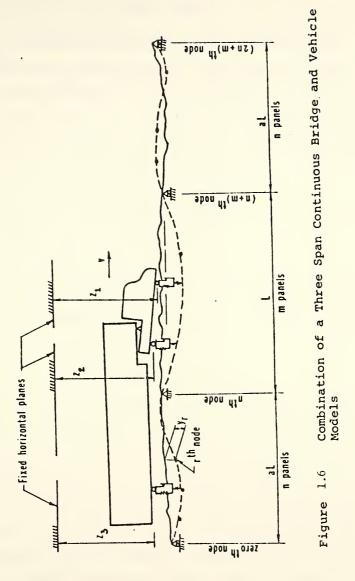


In this analysis the bridge is idealized as a single continuous beam and the resulting infinite degree of freedom system is replaced by a discrete system having a finite number of degrees of freedom. This discretization is effected by replacing the distributed mass by a series of concentrated point masses and considering the beam stiffness to be distributed as in the original structure. Bridge damping is assumed to be of the absolute viscous type, and is approximated by dashpots located at mass coordinate points. The analysis is based on ordinary beam theory, neglecting the effects of shearing deformation and rotary inertia.

Since the bridge has been idealized as a single beam, the rolling effect of the vehicle cannot be considered in this analysis. Even when treated as a plane system, however, a vehicle is a complex mechanical system. In this analysis a tractor-trailer vehicle is represented by a three-axle load unit consisting of two interconnected masses as shown in Figure 1.6. Each axle is represented by two springs in series and a frictional mechanism which simulates the effect of friction in the suspension system. The second spring is active only when the axle force exceeds the limiting friction value. Viscous damping is neglected.

Writing the equations of motion for the vehicle and the concentrated masses of the bridge yields a set of simultaneous, second-order differential equations, equal in number to the number of degrees of freedom of the bridge-vehicle system. These equations are solved by a numerical integration scheme in which the evaluation of the interacting forces between the bridge and the vehicle is a major intermediate step. As the integration of the differential equations is carried out, the values of all coordinates, accelerations, and interacting forces







are determined. Values of corresponding deflections and moments at any section can then be determined by statics.

1.4 ACCELERATION STUDIES FOR TWO SPAN CONTINUOUS BRIDGES

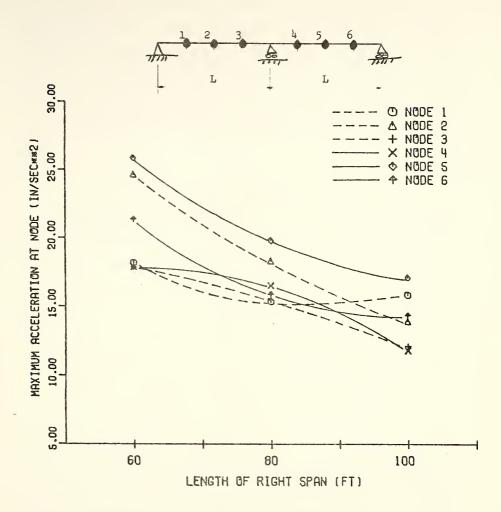
Both two and three span symmetric continuous girder bridges have been studied extensively (2); however because the results are similar only two span bridges will be discussed herein. Bridge data again has been taken from Reference (7) and the study is restricted to steel continuous beam bridges with reinforced concrete decks. Unless stated otherwise, the roadway surface is assumed to be smooth.

The accuracy of the analysis now depends on the number of lumped masses chosen as well as the number of integration steps. Good stability of the solution was obtained by dividing the time required for the vehicle to cross the bridge into 2000 integration steps. Following Huang's (8) suggestion, the bridge was modeled by lumping the masses at the quarter and midpoints of each span.

1.4.1 Bridge Parameters

In Figure 1.7 the maximum nodal accelerations are shown for three standard design bridge spans. The roadway width is 44 ft. and the 7-1/2 in. slab is supported by six rolled steel beams. Bridge damping is taken as 2 percent of critical damping. The vehicle is an HS20 three-axle truck moving at a speed of 60 mph. As with the simple span bridges, higher accelerations occur on the shorter spans. Maximum accelerations for two span bridges were found to be about 1.5 times larger than those of three span bridges of equal span length. Highest accelerations occur in the simple span bridges.





SPAN RATIO = 1.0 LEVEL SURFACE C/CR=0.02 N=4
HS 20-44 SMOOTH VEHICLE 60 MPH

Figure 1.7 Effect of Span Length on Acceleration (1 in. = .0254 m, 1 ft. = .305 m)



The effect of varying girder stiffness was investigated for a bridge with two 60 ft. spans. Although the results shown in Figure 1.8 are somewhat irregular, maximum accelerations for this particular bridge would not be significantly increased by replacing the A36 beams by smaller high strength steel beams. The important point here is, of course, that deflection control is not directly related to vibration control.

1.4.2 Vehicle Parameters

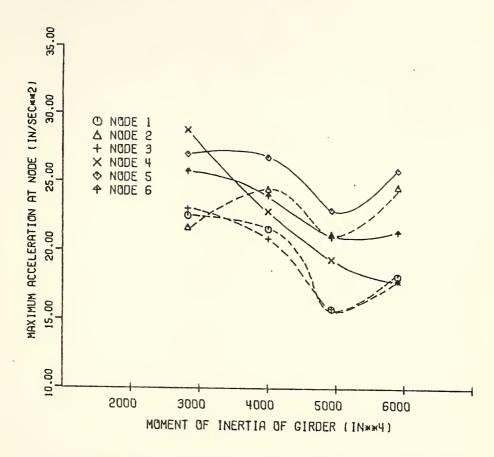
The geometric, mass and suspension parameter values of the vehicle model were chosen to make it closely represent a typical heavily loaded tractor-trailer. The spacing of the tractor axles was taken as 12 ft. but the spacing of the trailer axles might be considered variable. As shown in Figure 1.9, the maximum acceleration for two 60 ft. spans occurs with a trailer axle spacing of about 25 ft. or 0.42 times the span.

For three span bridges the critical axle spacing ratio is 0.37 to 0.43.

A comparison of maximum accelerations was also made using one, two and three axle vehicle models. Maximum accelerations were about the same for two and three axle vehicle models, but they were about two-thirds of the magnitudes produced by the single axle vehicle model. As was the case with simple span bridges, a marked increase in acceleration with vehicle speed was found.

Maximum accelerations for different values of frequency ratio, which is ratio of the natural frequency of the vehicle on its tires to the natural frequency of the bridge, were also computed. Generally, the magnitudes of accelerations at the nodes were about the same for all values of frequency ratio although the midspan accelerations were slightly higher when the vehicle and bridge had the same natural frequency.

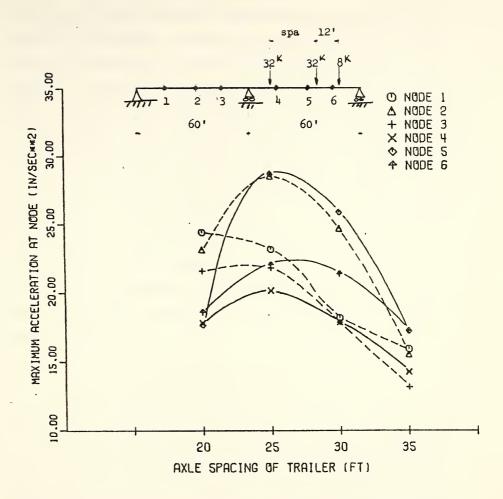




60-60 SPAN BRIDGE 6 GIRDERS LEVEL SURFACE C/CR=0.02 N=4 HS 20-44 SMOOTH VEHICLE 60 MPH

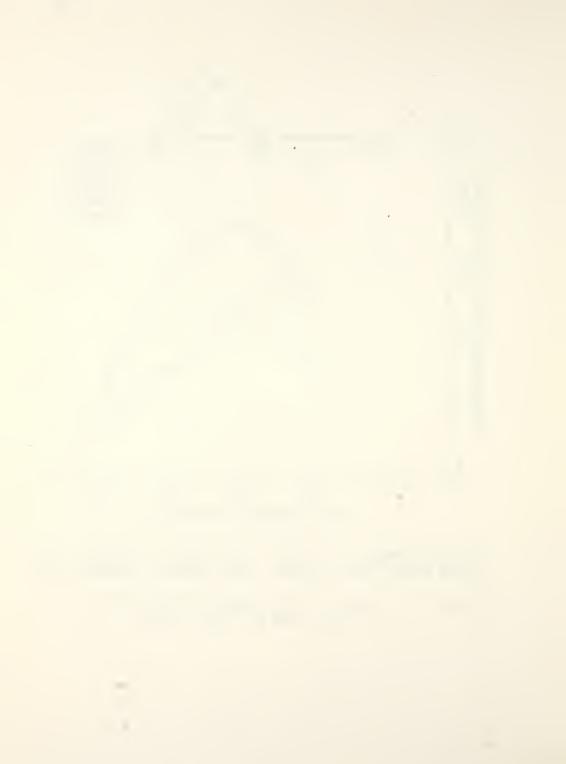
Figure 1.8 Effect of EI on Acceleration (1 in. = .0254 m)





60-60 SPAN BRIDGE 6 GIRDERS LEVEL SURFACE C/CR=0.02 N=4 3-AXLE SMOOTH VEHICLE 60 MPH AXLE SPACING OF TRACTOR = 12 FT

Figure 1.9 Effect of Axle Spacing on Acceleration (1 in. = .0254 m, 1 ft. = .305 m)

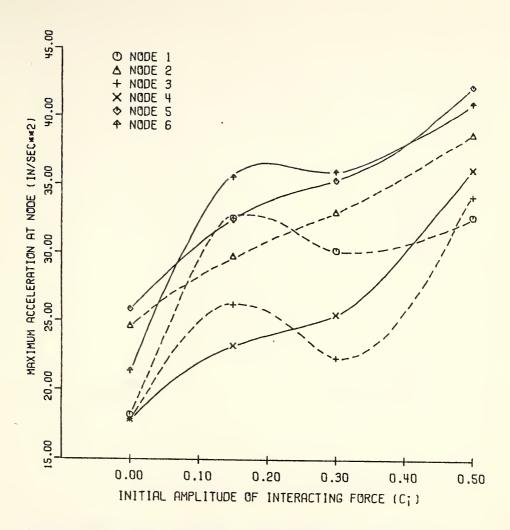


A real vehicle is likely to be oscillating somewhat as it enters a bridge due to approach pavement irregularities and a possible discontinuity at the abutment. The most significant parameter for represening initial oscillations is the amplitude of initial axle force variation $\mathbf{C_i}$. The initial axle force is equal to $(1+\mathbf{C_i})$ times the static force. Figure 1.10 shows maximum nodal accelerations for four values of $\mathbf{C_i}$. The same $\mathbf{C_i}$ value is assumed for each axle. According to Reference (9), $\mathbf{C_i}$ = .15 might correspond to a 1/8 in. pavement irregularity, and $\mathbf{C_i}$ = .50 might represent a large discontinuity at the abutment. It can be seen that the initial oscillation causes a 30 to 50 percent increase in maximum acceleration for this particular bridge-vehicle system, which is assumed to have a smooth deck surface. An investigation was also carried out considering a phase angle difference between initial oscillations of the trailer axles. Maximum accelerations varied less than 20 percent.

1.4.3 Roadway Roughness

All of the preceding two span studies have assumed a smooth bridge deck surface. The effect of surface roughness has been investigated for a standard design bridge having two 60 ft. spans and loaded by an HS20 truck. The deck surface is represented by an integral number of half sine waves. The very great influence of roadway roughness is shown in Figure 1.11. With four half waves of roughness in each span, the frequency of oscillation of the interacting forces is close to the fundamental natural frequency of the bridge and the nearly resonant response occurs. Thus for both simple and continuous bridge surface roughness seems to be the most significant factor affecting roadway accelerations.

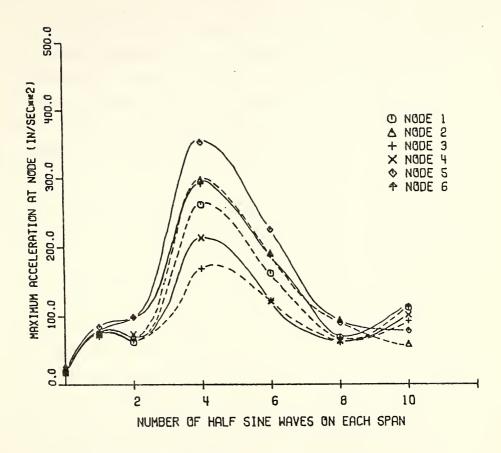




60-60 SPAN BRIDGE 6 GIRDERS LEVEL SURFACE C/CR=0.02 N=4 HS 20-44 OSCILLATING VEHICLE 60 MPH

Figure 1.10 Effect of Initially Oscillating Vehicle on Acceleration (1 in. = .0254 m)





60-60 SPAN BRIDGE 6 GIRDERS C/CR=0.02 N=4 HS 20-44 SM00TH VEHICLE 60 MPH ROUGH SURFACE

Figure 1.11 Effect of Surface Roughness on Acceleration (1 in. = .0254 m)



CHAPTER 2

EXPERIMENTAL STUDY

2.1 GENERAL

Although the dynamic responses of single-span and multi-span bridges to vehicular loadings have been the subject of several theoretical investigations in recent years, only a few experimental studies concerned with this important aspect of bridge behavior have been reported. The general objective of this phase of the research was to measure and document certain dynamic response characteristics of typical highway bridges.

The investigation previously reported in detail by Kropp (5) and summarized in this Chapter involved two general categories of activities:

- (a) field tests of a number of bridges, and
- (b) reduction and analysis of the collected data.

A sample of 62 bridges of various types of construction and span lengths was selected for testing. An instrumentation system was developed for obtaining key displacement and acceleration records corresponding to single-vehicle traverses by various types of vehicles sampled from normal traffic; this type of information was obtained for each bridge tested. In addition, more comprehensive tests were conducted under better-controlled conditions on three representative structures during the later stages of the testing program.

The acceleration and displacement data were recorded in analog form in the field and subsequently digitized to facilitate analysis. Data reduction and analysis involved the development of several computer programs and determination of maximum deflection, maximum velocity, maximum jerk and frequency content for each digitized vehicle-traverse record.



2.2 DESCRIPTION OF THE TESTING PROGRAM

The purpose of the testing program was to obtain field data which would adequately describe the dynamic response characteristics of a representative sample of existing highway bridges. Dynamic response information was collected for 62 bridges; the data were collected on-site in analog form on magnetic tape and reserved for subsequent digitization.

The data was collected in two phases. An initial survey phase,

Phase I, was carried out during the summer of 1973. The objective

of this phase of the work was to develop and check out instrumentation

and test procedures and to collect basic and preliminary information

on each of the bridges. Sixty bridges were involved in Phase I. The

second phase of the testing program, Phase II, was conducted during

the summer of 1976. Eighteen bridges were tested during this period,

sixteen of the Phase I group plus two others. More comprehensive data

collection and special testing were the primary purposes of this part

of the program.

2.2.1 Criteria and Procedures Used in Development of Bridge Sample

The bridges included in the sample were selected from a comprehensive listing of all bridges on state and interstate highways in Indiana. Categories of bridge types were established. Preference was given to types which have been constructed in greater numbers, especially those with similar configurations and physical dimensions. The ten categories of bridges included in this study are given in Table 2.1.

Each category was then broken down into sub-categories, each sub-category containing bridges having the same number of spans, span length, deck width, and, if possible, the same age (year of construction).



Table 2.1. Ten Categories of Bridges Selected for the Testing Program

Category Number	Type of Construction		
1	Non-Composite Steel Beam (Single- Span)		
2	Non-Composite Continuous Steel Beam		
3	Composite Non-Continuous Steel Beam		
4	Composite Continuous Steel Beam		
· 5	Composite Continuous Plate Girder (essentially uniform moment of inertia)		
6	Composite Continuous Plate Girder (non-uniform moment of inertia)		
7	Non-Continuous Reinforced Concrete Girder		
8	Continuous Reinforced Concrete Girder		
9	Continuous Reinforced Concrete Slab		
10	Prestressed Concrete I-Beam Bridge		



In all cases, bridges in a given sub-category had been designed in conformance to identical design standards.

The end result of the selection process was a sample consisting of 62 bridges; '40 with steel beams or girders as primary longitudinal super-structure elements, 19 with reinforced concrete superstructures, and 3 with prestressed concrete girders. The ten categories, which were based on type of construction, were divided into 22 sub-categories with two to four sub-categories per category. The sub-categories consisted of one to six similar bridges based on number of spans, span length, deck width, and year of construction. These four selection factors varied widely between sub-categories with the number of spans varying from one to four, the span lengths from 27 ft. to 129 ft., the deck width from 24 ft. to 51 ft., and the year of construction from 1929 to 1972.

Summarizing, the sample of bridges includes a diverse variety of bridge types, sizes, ages and traffic conditions. It contains small groups of essentially identical bridges, thus making it possible to identify common dynamic properties. The total sample of 62 bridges is sufficiently general to permit identification of general dynamic properties such as maximum accelerations and fundamental bending frequencies.

2.2.2 Test Variables

Some of the test variables were:

- (1) speed of the vehicle crossing the bridge,
- (2) total weight of the vehicle
- (3) type of vehicle
- (4) vehicle parameters, such as spring stiffness and natural frequency of the system of tires and springs,



- (5) transverse location of the vehicle on the bridge, and
- (6) initial conditions of the vehicle as the vehicle entered the bridge.

Except for the special tests, discussed in a following section, none of the above variables were subject to direct control.

Vehicle codings - see Table 2.2 - were devised for each of the study phases to permit identification of the type of vehicle associated with each vehicle crossing record. The fairly detailed coding categories utilized for Phase II testing afforded a means for estimating vehicle lengths and axle weight ranges through reference to existing loadometer studies.

During each test, the test equipment and personnel were positioned so as to minimize any influence their presence would have on normal traffic flow.

Test Vehicle

Even though one of the main objectives of the testing program was to record responses for a broad range of bridge structures under normal traffic loadings, it was decided that a reference test vehicle with predetermined key vehicle parameters should also be used. It was necessary that these parameters be known for at least one vehicle if meaningful comparisons between analytical predictions and experimentally measured responses were to be made. Moreover, for bridges with low heavy-vehicle traffic it was found that using the test vehicle was the only practical way to obtain significant dynamic response records.

The reference test vehicle used was a school bus owned by the Indiana State Highway Commission's Research and Training Center. This bus, a 1969 International with dual rear wheels and 23 ft. wheel base,



Table 2.2. Vehicle Classification Codes for Phase I and II Testing

	Code	Vehicle Description
Phase I		
	1	Passenger Car
	2	Pick-ups to 2-ton trucks
	3	Tractor-trailers, Dumptruck, etc.
	4	Test Vehicle (Research & Training Center Bus)
Phase II		
	1	Passenger Car
	2	Test Vehicle (R & TC Bus)
	3	Pick-up/light delivery vans
	4	Two axle truck
	5	Three axle truck
	6	Two axle tractor-one axle trailer
	7	Two axle tractor-two axle trailer
	8 .	Three axle tractor-one axle trailer
	9	Three axle tractor-two axle trailer



was loaded so that the gross vehicle weight was 21,000 lbs. (6760 lbs.

-front axle, 14240 lbs.-rear axle). Vehicle parameters measured in addition to axle weights were axle spacing, front and rear spring stiffnesses, and front and rear tire stiffness. A summary of the more significant characteristics of this vehicle is given in Table 2.3.

2.2.3 Instrumentation and Testing Procedures

Acceleration was the physical quantity of primary interest since this motion attribute appeared to be the one most significantly related to human sensitivity to vibration. Accelerations were measured at midspan (or near-midspan) locations on opposite sides of each bridge deck; for multiple-span bridges these measurements were made for each span. With these data it was possible to identify and analyze both fundamental flexural-mode and torsional-mode responses. Previous studies have concluded that highway bridges vibrate primarily in these modes.

Deflection response was also measured at a single location for each bridge. The deflection gage was installed adjacent to an accelerameter so that corresponding acceleration and deflection records could be generated for the same point. The data thus produced provided a basis for development of data analysis techniques and procedures which are subsequently described.

The testing program involved collection of substantial quantities of data. All records of bridge motion were recorded directly and permanently in analog form on instrumentation-quality magnetic tape. This approach to data collection was selected because of its relative economy, simplicity, and flexibility.



Table 2.3. Summary of Test Vehicle Parameters*

Vehicle Type: 1969 International School Bus

23.0 ft. wheel base with dual rear wheels

Axle Weights: Front Axle 6760 lbs.

Rear Axle 14240 lbs.

Gross Vehicle Weight 21000 lbs.

Spring Constants (lbs/in):

	Front	Rear
Tires, k _t	6153.	13668.
Springs, k _s	3391.	6273.

Pseudo Frequencies (cps):

,	Front	Rear
If only tire spring acts, f _{t,i}	2.98	3.06
If tire and suspension system act in series fts.i	1.78	172

*Note. The values given are for Phase II testing. The values for Phase I are identical for all practical purposes.



Test Apparatus

The following is a brief description of the major pieces of equipment used to collect and store the data. More complete information concerning these instruments is available elsewhere (5).

Kistler Model QA-116-15 accelerometers were used to measure vertical accelerations at selected points (usually mid-spans) as the selected vehicles traversed the bridge. This self-contained instrument can be utilized to measure steady state and moderate frequency accelerations up to 15 g's over a wide range of environmental conditions.

The deflection gage consisted of a cantilevered aluminum plate mounted on a block of concrete; electrical resistance strain gages were bonded to the cantilevered plate. The free end of this plate was connected to the bridge superstructure by a tensioned wire. Thus, the bridge deflections at the point of attachment were simultaneously reproduced by the cantilever beam. Deflection of the cantilever beam produced proporational strains in the gages which, in turn, were converted by means of appropriate signal conditioning into an electrical signal proportional to the deflection of interest.

Tandberg Series 100 Instrumentation Tape Recorders were used to record the outputs from both the accelerometers and deflection gages, as well as information concerning the vehicle type and the position of the vehicle on the bridge. Each recorder had the capacity to simultaneously collect data from four separate inputs and record these data on 1/4 in. magnetic tape.

Instrumentation Scheme

The instrumentation scheme for Phase II testing, which was developed in light of the experience gained from Phase I testing and data



reduction activities, is described below. The scheme used for Phase I testing was not greatly dissimilar.

In keeping with the objectives of this test program, each bridge tested was instrumented with one deflection gage and from two to eight accelerometers, depending upon the number of spans. For most tests two accelerometers were placed at midspan of each span, one on either side of the bridge, and attached to the curb. The deflection gage was always positioned adjacent to an accelerometer in the first span. For bridges over water it was not always possible to install the deflection gage exactly at midspan of the first span. For these cases the deflection gage was placed as near to mid-span as possible and the two accelerometers were placed at the same longitudinal position as the deflection gage. Figure 2.1 shows typical transducer locations for bridges having from one to four spans.

The output signals from the accelerometers and deflection gage were recorded in analog form at 3-3/4 in./sec. on 1/4 in. x 1800 ft. magnetic tape, using from one to three 4-channel tape recorders, depending on the number of accelerometers used. One tape recorder was sufficient for a single span bridge instrumented with two accelerometers and one deflection gage. For a four span bridge, eight accelerometers and one deflection gage were used, requiring three tape recorders.

Channel 4 of each tape recorder was reserved for information describing the type of vehicle traversing the bridge and the position of this vehicle with respect to the bridge. The type of vehicle was recorded as voltage pulses manually generated by means of a digital encoding device. Signals from air-activated switches coupled to



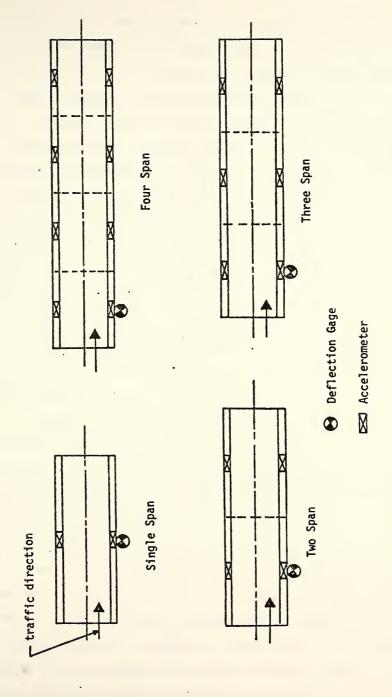


Figure 2.1 Accelerometer and delfection gage location for Phase I and II testing.



traffic hoses were used to indicate the longitudinal position of the vehicle with respect to the bridge. Since the air-activated switches were common to Channel 4 of each tape recorder, the signals from these switches provided a common time base among the data tapes.

Three traffic hoses were used. The first hose was attached to the roadway surface approximately 100 ft. in front of the entrance abutment; the second (entry) hose was attached to the approach slab adjacent to the initial (entrance) expansion joint. The third (exit) hose was fastened to the approach slab adjacent to the last (exit) expansion joint. As a vehicle approached the bridge, when the first axle passed over the first traffic hose a voltage pulse was recorded on Channel 4 of all active tapes. As the first axle passed over the remaining two traffic hoses, corresponding voltage pulses were recorded on all tapes. Approximately 10 seconds after the vehicle had tripped the exit hose the vehicle-type code was manually entered and simultaneously recorded on Channel 4 of all tapes.

A sample of the output from Channels 1, 2, 3 and 4 for a vehicle crossing a two span bridge can be seen in Figure 2.2. Note the voltage pulses on Channel 4 singifying vehicle position and vehicle type code.

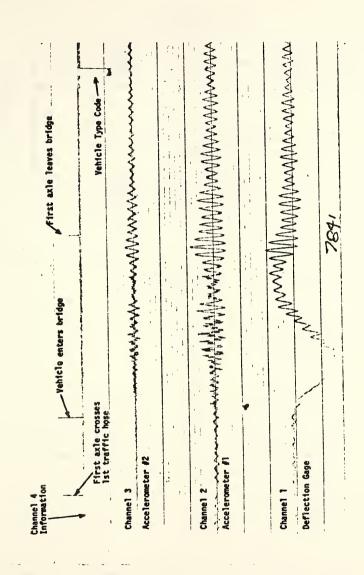
Also, see Figure 2.3 for a schematic diagram of the instrumentation for a two span bridge.

Test Procedures

The Indiana State Highway Commission's Research and Training Center carried out the data collection for both Phase I and Phase II testing.

The typical bridge-testing crew consisted of two engineers and four engineering assistants. The testing of a bridge usually took from half a day to two days, depending upon the type of traffic using the bridge and the distance between the bridge site and the Research and Training Center.





Portion of an Oscillographic Strip Chart Showing the Outputs of Channels 1, 2, 3, and 4 for a Vehicle Crossing a Two Span Bridge - Phase II

Figure 2.2



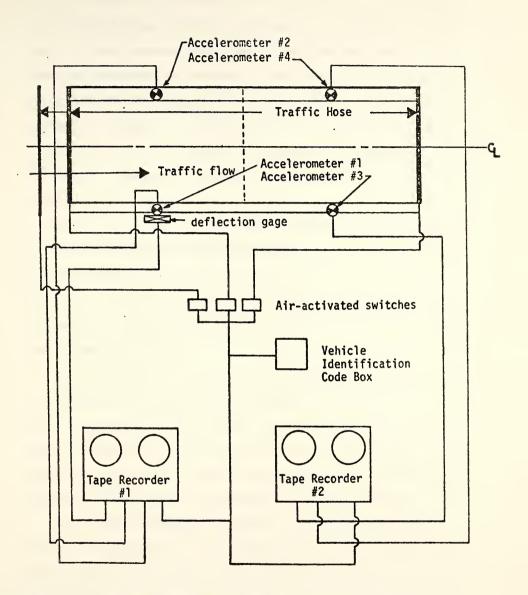


Figure 2.3 Schematic of Phase II instrumentation for a two span bridge.



All equipment and instruments were transported to the bridge site in a mobile laboratory, which was equipped with a portable AC generator. The accelerometers were bolted to stainless steel base plates which had previously been attached to the bridge curbs with epoxy cement. The deflection gage was placed below the first span and connected to the bridge curb with a tensioned wire. The traffic hoses were secured to the roadway and all transducers and information switches were connected to the appropriate channels of the tape recorders.

Before the actual testing was begun, information was gathered concerning the physical characteristics and structural condition of the bridge. Physical characteristics of the bridge recorded included sketches of the bridge summarizing all significant physical dimensions. In general, any condition which might affect the motion of the bridge was documented at this time.

During the test of a bridge all test personnel were stationed off the bridge and its approaches; when possible, crew members were positioned so as to be hidden from view. Only one person was stationed near the bridge in position to observe the approaching traffic. This "vehicle spotter" was in constant voice communication with the test personnel in the mobile lab operating the recording equipment.

As a vehicle approached the bridge under certain conditions which are described below - the vehicle spotter, if he wished the vehicle crossing to be recorded, activated the traffic hoses by entering
the proper code on a hand-held encoder. When the front wheels of the
vehicle triggered the first traffic hose an initial voltage pulse was
recorded on Channel 4 of each active tape recorder. As the vehicle
traversed the bridge, signals corresponding to each transducer were



recorded on their respective channels. Voltage pulses were also recorded on Channel 4 of each recorder as the vehicle front wheels triggered the bridge-deck entry and exit hoses. After the vehicle had left the bridge and a minimum of ten seconds had elapsed the spotter entered coded vehicle-type identification data by means of the encoder. The encoder was equipped with an automatic time delay feature which prevented premature entry of the vehicle code. There was no information entered on the records to indicate the lane occupied by the vehicle. A typical output record, as reproduced by means of an oscillographic recorder, is shown in Figure 2.2. The record shows all four channels of information and explanatory annotations.

It was stated previously that vehicle crossing records were collected only when, in the judgement of the vehicle spotter, certain criteria were met, i.e. when conditions were favorable for a satisfactory and necessary record to be obtained. Since it was intended that the measured responses be compared with analytical predictions, it was necessary that the bridge be relatively quiescent immediately prior to vehicle passage and that responses be determined for individual vehicles rather than vehicle groups. Since free vibration immediately after vehicular passage was also of interest, it was necessary that there be an elapsed period of not less than approximately 15 seconds between exit of the vehicle of interest and entry of a following vehicle. Finally, the collection of vehicle records for a given bridge had to represent a reasonable sample of the total traffic population; thus, a balancing of the number of records of each type was also a criterion.

Test Vehicle Procedures

Testing each bridge with the "test vehicle" described in Section 2.3 was integrated with the collection of data for mormal traffic.



When traffic was light no significant problems were encountered. Under heavier traffic conditions it was usually difficult to satisfy the above-stated criteria for collection of valid records for the test vehicle.

Generally it was possible for bridges carrying lightly traveled roads (e.g., county roads over an interstate highway) to collect records for crossing speeds of 10 mph, 20 mph, 30 mph, 40 mph, and sometimes 50 mph; at least two test runs, one in each direction where possible, were normally made for each speed. On higher volume conventional highways (e.g. state highways) test speeds of 40 mph and 50 mph were used, including repetitive runs and runs in different lanes. On bridges carrying interstate traffic a minimum of three runs at 50 mph in the right lane were conducted.

Special Tests

After the conclusion of the Phase II program a series of special tests were conducted on three bridges selected from the Phase II sample. These tests were carried out in order to obtain more complete data required for checking both data analysis and analytical (response prediction) computer programs.

These special tests were characterized by close control over test vehicle speeds and transverse positions on the bridge deck, and by more precise controls over other factors and conditions which would influence dynamic response. Quantitative data on deck roughness was also obtained for the bridges used for these special tests.



2.3 DATA REDUCTION AND ANALYSIS

In the course of this work more than 13,000 deflection and acceleration records corresponding to over 2200 vehicle crossings were collected. Cost and time constraints precluded reduction and analysis of all these data. Thus, only 900 vehicle crossing records were selected for analysis; of these approximately 65% were for trucks, 30% were for the test vehicle, and 5% were for various light vehicles.

Data reduction involved digitization of the analog (continuous) response records and storage of this information on magnetic tape in a format suitable for analysis. Reduction and analysis of the data were directed toward determination of certain key response characteristics - deflection, velocity, acceleration, and jerk maxima - frequency content of the response records, and damping characteristics.

2.3.1 Digitization of the Data

Since the response records were collected in analog form and the data analysis was to be carried out using the Purdue University Dual CDC 6500 digital computer facility, digitization was the requisite initial step in the data reduction process. Digitization of the records was carried out using equipment made available by Professor Anshel Schiff of the School of Mechanical Engineering, Purdue University.

The system used for digitizing and storing the data consisted of one of the Tandberg tape recorders used in collecting the field data; a 100 Hz low-pass filter; a data acquisition system (DAS), which included a 9-track tape drive; a disk system; a line printer; and a CRT terminal with keyboard and display. A block diagram of the equipment set-up may be seen in Figure 2.4.

Control of the digitization process was maintained through operator-directed use of software which was developed specifically



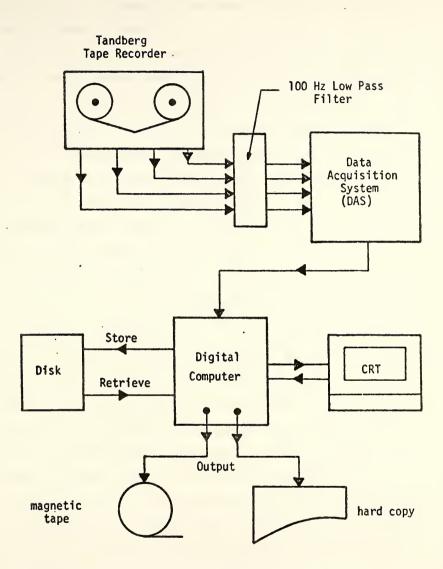


Figure 2.4 Equipment set-up for digitizing the data.



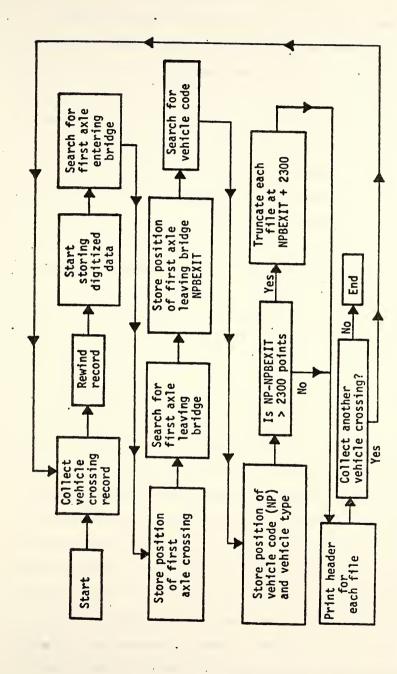
for this work. A flowchart for the Phase II digitization computer program is shown in Figure 2.5.

Prior to initiation of the digitization process for a given vehicle crossing record (hereinafter called record) the sampling interval for the DAS was set at 5 milliseconds. Utilizing this sampling rate of 200 points per second on data previously screened by a 100 Hz low-pass filter effectively removed, without aliasing or redundancy, all information with frequencies greater than 50 Hz. This 50 Hz cut-off frequency is well above the level of the highest significant response frequency encountered in previous analytical and experimental studies of highway bridges.

To initiate the digitization of a given vehicle crossing record, information describing the record was entered through the CRT terminal for storage on the digitized tape. Next, the four data channels on each analog tape were digitized simultaneously by the DAS and temporarily stored on disk in accordance with the format prescribed by the control program. The digital mini-computer then processed all four channels of information and stored these data files on 9-track magnetic tape; each file (i.e. output information obtained from a single transducer) was assigned a distinct consecutive file number by the computer. The final step in the digitization process was conversion of the digitized records from IBM 9-track EBCDIC to IBM 7-track BCDIC, which was compatible with the Purdue University Computer Center CDC 6500 system's input capabilities.

The original (analog) data tapes and the digitized versions of these vibration records have been stored in the event that additional data reduction and/or analysis is found to be desirable.





Flowchart for the Phase II digitization computer program. Figure 2.5



2.3.2 Analysis of the Test Data

Studies of human perception of bridge motion have generally concluded that comfort criteria for pedestrians and passengers in halted vehicles can be related to magnitudes of deflection, velocity, acceleration, and jerk. The analysis of the considerable volume of experimental data collected in the course of this research was arbitrarily limited to determination of maximum values of each of these response quantities, determination of the intrinsic damping characteristics, and spectral (frequency) analysis of both the forced and free vibration parts of each vehicle crossing record. Maximum, i.e. peak, values of deflection, velocity, acceleration, and jerk were thus adopted as reasonable indices of overall bridge response to a traversing vehicle. The frequency analysis involved determination of Fourier spectra for the forced and free vibration portions of the record. An apparent equivalent viscous damping ratio was inferred from the free vibration portion of each record.

As described previously, response records collected for each bridge included the outputs of accelerometers and a single deflection gage, all placed at or near midspan locations. The maximum indicated acceleration for each vehicle crossing record was determined by a (computer program directed) systematic search of all data files containing the outputs of accelerometers. The peak value indicated during a given vehicle traverse by the single deflection gage was determined; this maximum total (i.e. static plus dynamic) deflection measured at the site of the deflection gage is called the maximum deflection herein.

Since neither velocity nor jerk was measured directly by transducers, it was necessary to obtain these quantities indirectly. To accomplish this algorithms were developed which produced velocity and



jerk response histories using the digitized acceleration and deflection response records corresponding to the two transducers located at the same point on the bridge deck. Specifically, algorithms were developed which: integrate an acceleration file (response record) once and twice to obtain the corresponding velocity and deflection files (response histories); differentiate an acceleration file to obtain the corresponding jerk file; and differentiate a deflection file once, twice, and three times to obtain the corresponding velocity, acceleration, and jerk files.

Certain problems are associated with using a raw data file containing a digitized response history of a particular function to obtain functions which are derivatives or integrals of the measured function. If a raw data file is differentiated directly the inherent random noise in the digitized data is significantly magnified by differentiation and the results are generally unsatisfactory. The fact that the reference axis of zero acceleration (the "true axis") is generally unknown due to non-zero and indeterminate initial conditions produces so-called baseline errors in the velocity and deflection histories obtained by direct successive integration of the raw acceleration data file.

To obtain satisfactory results by differentiation it was first necessary to describe, point-by-point, the data using so-called "smoothing polynomials". These polynomials were obtained by a least-square point-by-point fitting using a specified number of data points either side of the point of interest, of a polynomial of specified (second through fifth) degree. Determination of optimum smoothing polynomial parameters for each data set was essential, since excessive smoothing results in loss of valid data and



insufficient smoothing leads to spurious results. (These parameters are: J, the specified degree of the smoothing polynomial; and K, the index describing the number of consecutive data points centered on the point of interest to be used in establishing the best least-square fit). Specific parameter combinations were selected for the various data analysis tasks based on the results of extensive comparative studies.

Obtaining satisfactory velocity and displacement records by integrating acceleration records can be accomplished only by applying suitable corrections to the digitized acceleration file. Seven different types of baseline corrections were investigated and tested. As a result of these studies, distinct corrections best suited to each of the typical bridge types (single span, two span continuous, and three span continuous) were identified and used in analyzing the data.

Spectral analysis was used to determine the character of the frequency content of the recorded acceleration and displacement response records. Using the results of this analysis it was possible to establish which response frequencies and modes were most significant for each record. This information has considerable potential value for researchers attempting to develop analytical models for response prediction. In addition, natural frequencies established from these analyses were used to infer bridge stiffness properties.

For purposes of spectral analysis, each data file (response history from a transducer due to an individual vehicle crossing) was subdivided into two parts: the forced response of the bridge due to vehicle/bridge interaction, i.e. that portion of the response history recorded while the vehicle was on the bridge; and the post-traverse



free vibration of the bridge, i.e. that portion of the response history recorded immediately after the vehicle moved off the bridge. A Fourier spectrum was then calculated for each of the two parts of each file by utilizing Fast Fourier Transform procedures.

A measure of the damping characteristics of each bridge was acquired by calculating the logarithmic decrement and equivalent viscuous damping ratio for the free vibration part of each of the response records and determining average values for the damping ratio from these results.

Comprehensive computer programs were written to facilitate reduction and analysis of the test data. The programs were designed to read the digitized data files directly from the 7-track BCDIC magnetic tapes, process each file, and plot the results. The following functions were performed by these programs:

- (1) calculate the calibration constants.
- (2) find the log decrement and damping ratio of the bridge using both deflection and acceleration files,
- (3) smooth the deflection file and differentiate it either once, twice or three times for the corresponding velocity, acceleration and jerk, respectively,
- (4) smooth the acceleration file (from Accelerometer #1 only) and calculate the corresponding jerk by differentiation,
- (5) integrate the acceleration file either once or twice to obtain the corresponding velocity or deflection file.
- (6) compare the deflection from the twice integrated acceleration with the actual deflection,
- (7) compare the acceleration from the twice differentiated deflection with the actual acceleration,

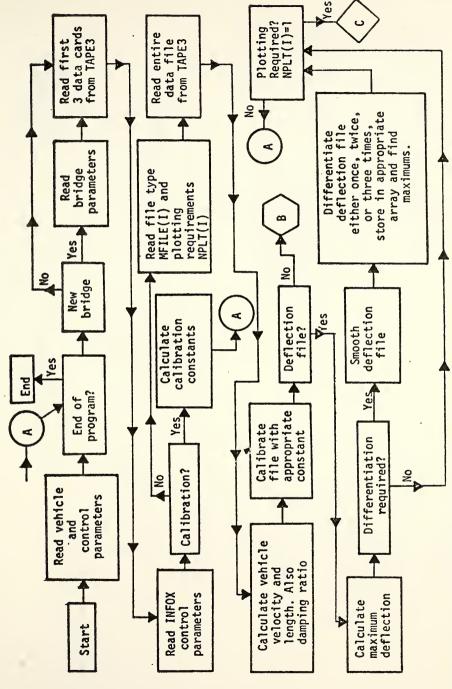


- (8) calculate the maximum values for deflection, velocity, acceleration and jerk,
- (9) calculate the vehicle velocity and identify vehicle type,
- (10) plot the deflection, velocity, acceleration, or jerk file for the portion of the file when the vehicle was on the bridge,
- (11) plot the actual deflection versus the deflection from twice integrated accleration,
- (12) plot the actual acceleration and acceleration from twice differentiated difference between the two quantities,
- (13) plot the acceleration files and their difference from two accelerometers located in the same span but on opposite sides of the bridge, and
- (14) calculate and plot the Fourier Spectrum for each file.

 Figures 2.6 and 2.7 are flowcharts for the programs which were used for data reduction and analysis.

Typical results of the data analysis are shown in Figures 2.8 through 2.19. Figures 2.8 through 2.13 are examples of the results of the integration procedures which were utilized to obtain displacement response histories from acceleration data; note that these figures show the character of the measured and inferred deflection histories and the differences between them. Figures 2.14 through 2.16 present sample results obtained by smoothing a deflection file and differentiating twice to obtain an acceleration response history; the results are presented in a similar comparative format. Figures 2.17 through 2.19 are examples of frequency spectra obtained using the computer program described by Figure 2.7.





Flowchart for Phase II data reduction computer program. Figure 2.6



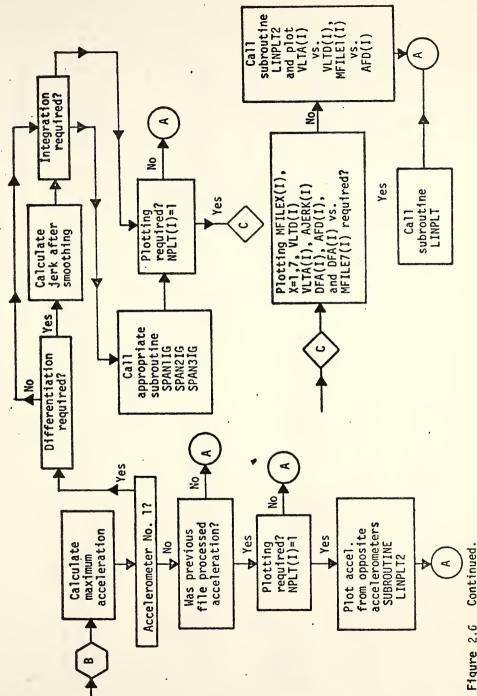
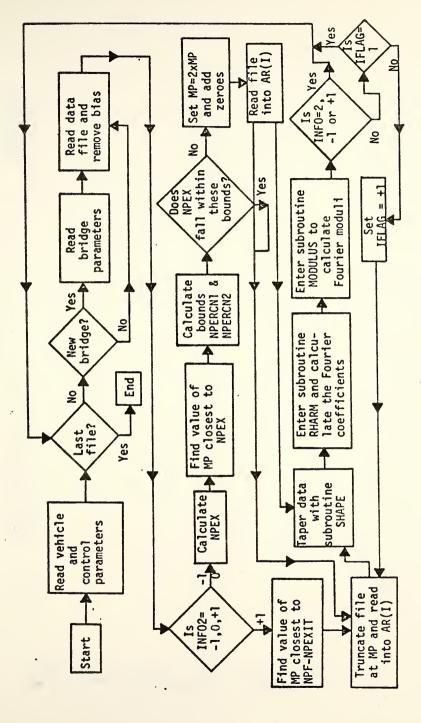


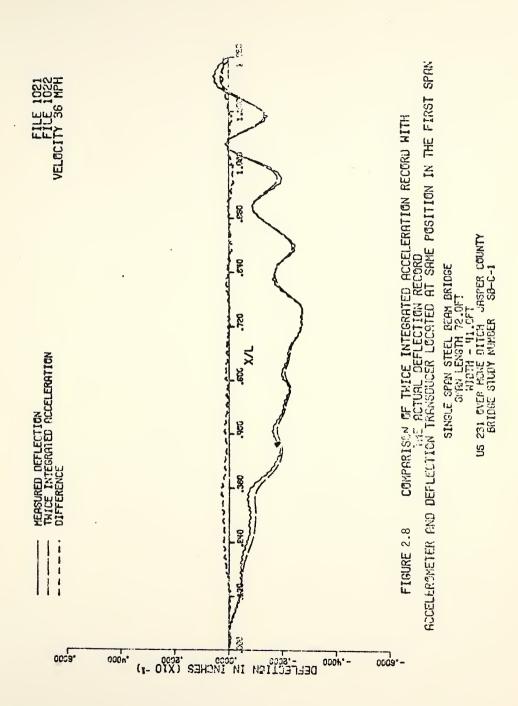
Figure 2.6



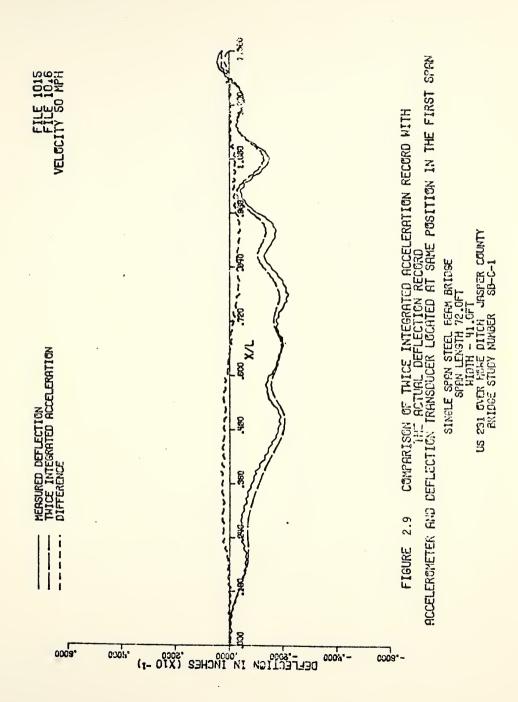


Flowchart for the computer program used for calculating the Fourier Spectrum. Figure 2.7

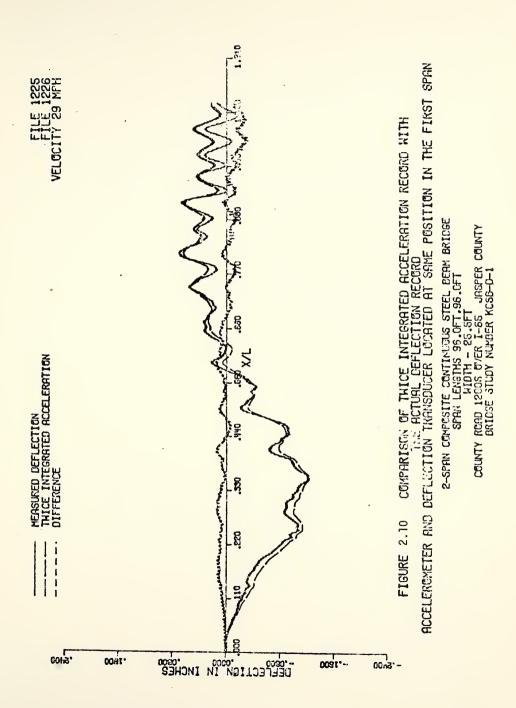




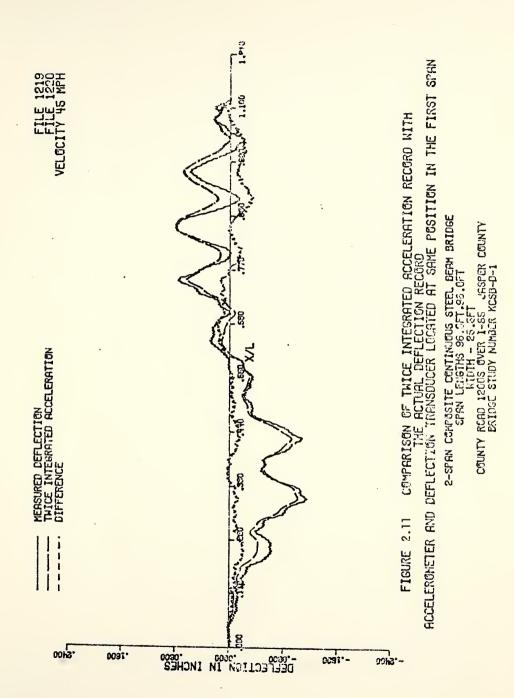




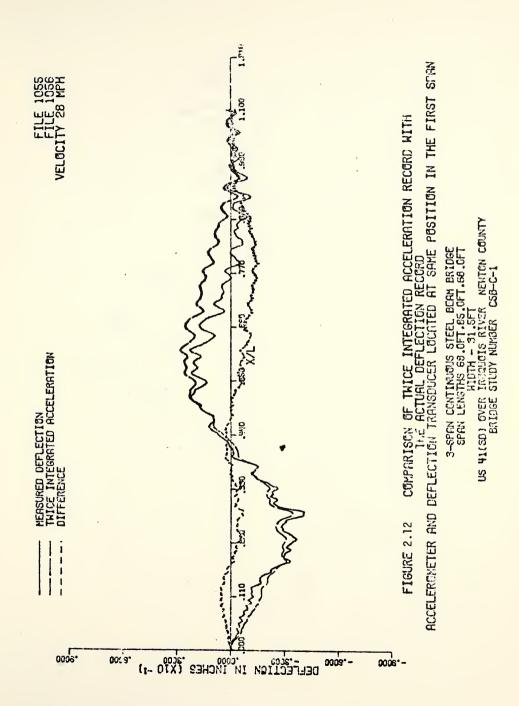




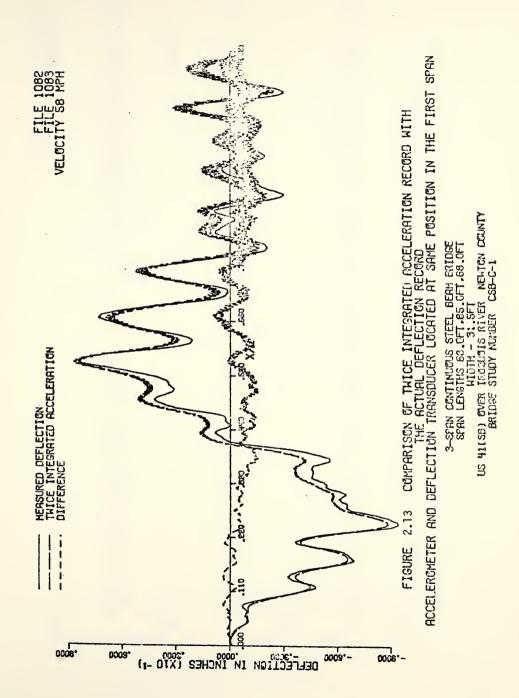




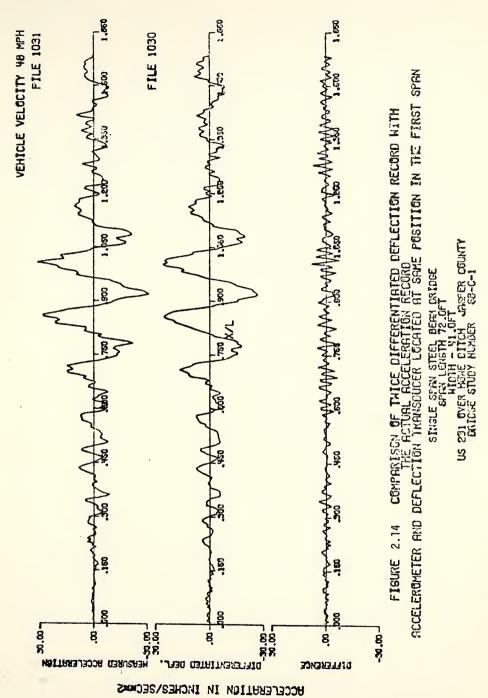






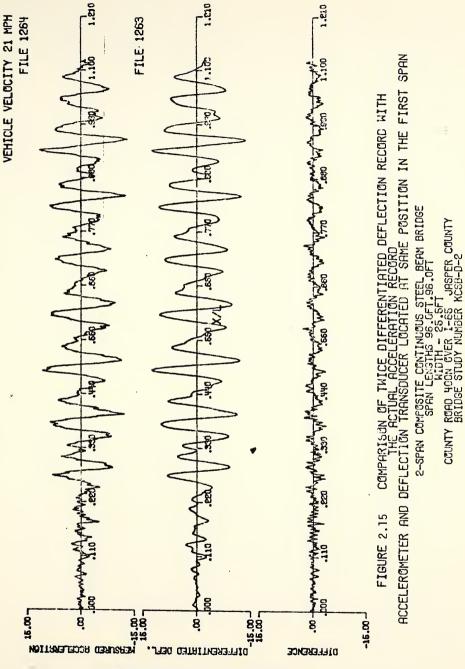






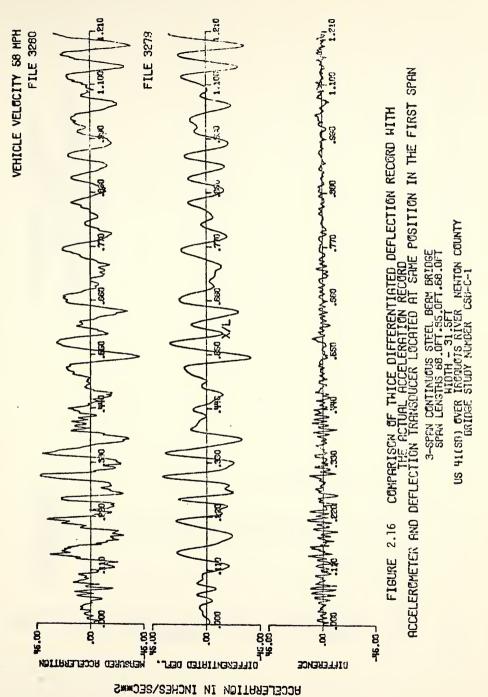
ACCELERATION IN INCHES/SECKING



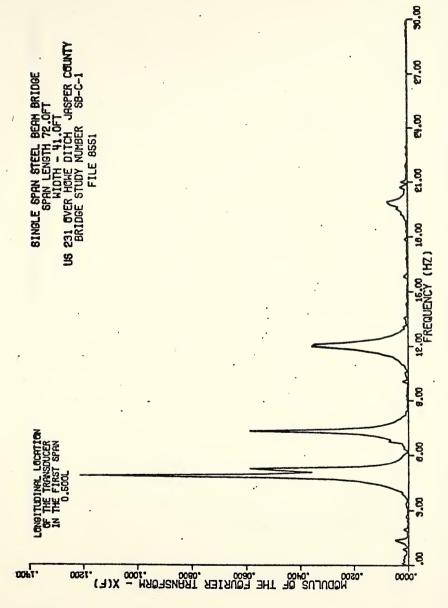


ACCELERATION IN INCHES/SECKING



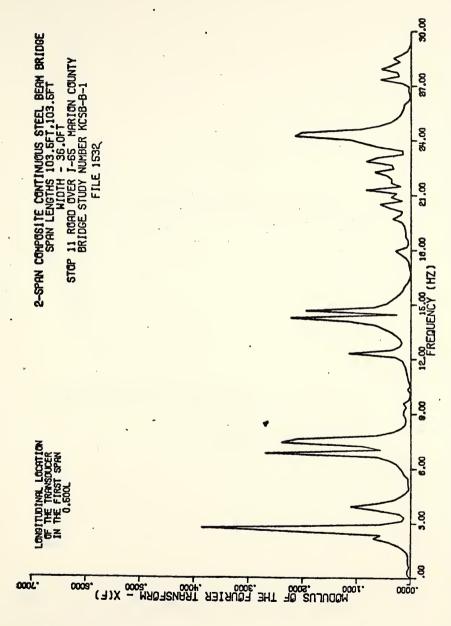






Fourier Spectrum of the Free Vibration of Acceleration, Transverse Vehicle Position-Travel Lane. Figure 2.17

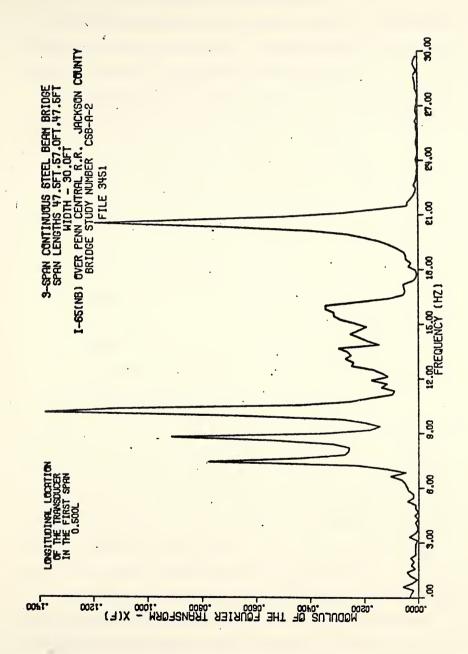




Fourier Spectrum of the Free Vibration Portion of Accelerometer No. Figure 2.18



2



Fourier Spectrum of the Free Vibration Portion of Accelerometer No. Figure 2.19



2.4 SUMMARY OF THE TEST RESULTS

An abbreviated summary of the principal results of this experimental study is given in this section. A complete documentation of the results is available in the report authored by Kropp (5).

Each digitized vehicle crossing record, was analyzed using the procedures and computer programs described in the previous section.

The following response characteristics were determined:

- (1) maximum deflection from the deflection gage data,
- (2) maximum acceleration indicated by the data from each accelerometer,
- (3) damping ratio from the free vibration portion of the data from each transducer,
- (4) maximum velocity and deflection from integrated acceleration data,
- (5) maximum velocity, acceleration, and jerk from smoothed and differentiated deflection data, and
- (6) maximum jerk from differentiated acceleration.

The major results of this analysis are presented in Table 2.4. The results given in this table are limited to test results for vehicles falling in the heaviest class (three axle tractor - two axle trailer), since these vehicles produced the most significant responses. Results from only six of the ten bridge categories are included in this table; there was insufficient heavy-vehicle traffic on the bridges in the other four to warrant presentation here of those results.

The absolute maximum and mean maximum (i.e. the mean value of the population of all maxima, without regard to sign, determined from all digitized heavy-vehicle crossing records for the bridge of interest) values given in Table 2.4 were determined for the following



response parameters for each of the indicated bridges: deflection, velocity, acceleration, and jerk. Note that the maximum deflection shown is Table 2.4 is the largest numerical value measured (plus (+) up and minus (-) down) and that the maximum acceleration presented is also the largest value encountered in the record without regard to sign. The maxima for deflection, velocity, and jerk are those values observed (for deflection) or inferred (for velocity and jerk) for the one point on the bridge instrumented with both the deflection gage and an accelerometer; the maximum for acceleration is the largest value measured by any accelerometer on the deck. (The maximum acceleration did not consistently occur at any particular position for any of the bridges tested).

Steel bridges exhibited generally higher responses (acceleration levels were about twice as large) than those of reinforced or prestressed concrete. Responses as measured by mean maximum acceleration were generally similar for the different types of steel bridges tested. Differences between steel and concrete bridge motion as measured by jerk are less pronounced but steel bridge jerk values are somewhat larger. Jerk values generally vary inversely with the span length.

The levels of acceleration recorded were generally well within acceptable ranges when compared to currently employed comfort criteria, which are discussed in a subsequent chapter of this report. The highest measured acceleration peak was 133 in/sec²; there were only five instances in the entire testing program where a single vehicle crossing produced an acceleration larger than 100 in/sec². The highest mean maximum acceleration (for all recorded vehicles crossing a bridge) was only 74 in/sec². The type of bridge exhibiting these largest observed, but still tolerable, responses was the two-span continuous composite steel type.



Table 2.4 Summary of Test Results for Heaviest Vehicle Class

Absolute/Mean Maximum Jerk (in./sec ³)		7760 5080	2320 1490		7790 5110	4580 2900	5910 2740		NA NA
Absolute/Mean Maximum Acceleration (in./sec ²)		89 61	50 33		105 53	123 58	83 52		133 74
Absolute/Mean Maximum Velocity (in./sec)		1.38	1.16		1.55	1.40	1.84		NA NA
Absolute/Mean Maximum Deflection (in.)		-0.060 0.021	-0.058 -0.031		+0.143 0.027	-0.075 0.041	-0.137 0.082		-0.228 0.099
Span Length(s) (ft)		09	72		47.5-57-47.5	60-72-60	68-85-68		76–76
Bridges	Category 1	SB-A-1 through SB-A-5	SB-C-1	Category 2	CSB-A-1 through CSB-A-4	CSB-B-1 through CSB-B-4	CSB-C-1	Category 4	KCSB-C-1 through KCSB-C-4



Table 2.4 Summary of Test Results for Heaviest Vehicle Class (Continued)

Absolute/Mean Maximum Jerk (in./sec ³)		4300 3150	1440 1150		1280 700		3280 2180
Absolute/Mean Maximum Acceleration (in./sec ²)		45 24	59 25		30 16		41
Absolute/Mean Maximum Velocity (in./sec.)		0.43	0.21 0.15		0.15 0.10		0.89
Absolute/Mean Maximum Deflection (in.)		-0.014 0.007	-0.025		-0.008		-0.066
Span Jength(s) (ft)		34-34-34	36-36-36		27-36-27		70-72-72-70
Bridges	Category 7	RCB-A-1 through RCG-A-3	RCB-B-1 through RCG-B-4	Category 9	CRCS-A-1 through CRCS-A-6	Category 10	PCIB-A-1 and PCIB-A-2



As cited previously, there were a number of significant factors influencing the motions observed for the bridges tested which were not subject to control during this program. Nevertheless, the conditions encountered were believed to be representative and typical; thus, the indices to response which were measured and determined may reasonably be assumed to be representative and typical.

Table 2.5 contains damping ratios determined for most of the categories and sub-categories of bridges tested. The values obtained in this study agree generally with those observed by other investigators (12,14).

Spectral analyses of the vehicle deflection and acceleration records resulted in frequency spectral plots like those shown in Figures 2.17, 2.18, and 2.19. More complete plots are shown in Appendix H of the original report (5).

The results of the spectral analysis and extensions therefrom are summarized in Table 2.6. To facilitate the identification of the fundamental bending frequency and the calculation of the percentage of composite action exhibited by the steel beam bridges, a theoretical fundamental (bending) frequency was calculated using the properties of the bridge cross section. For steel bridges designed for composite action and for concrete bridges, the calculated stiffness reflected the contribution of the bridge slab acting compositely with the beams or girders. For decks not designed for composite action, the stiffness was calculated as the sum of the beam-girder stiffness and the slab stiffness; i.e., the zero composite action stiffness was also evaluated.

Using the measured and calculated information discussed above it was possible to infer the relative degree to which composite action was actually realized in each structure. The results showed that the



Table 2.5 Typical Damping Ratios

Bridge Type	Damping Ratio
Single Span	
SB-A	2.0%
\$B-B \$B- C	2.2%
38-6	1.4%
Two Span	
KCSB-A	1.1%
KCSB-B KCSB-C	1.1%
KCSB-D	1.5% 1.4%
KCSG-A	
KCSG-B	1.2%
ACDC V	
KCPG-A KCPG-B	1.2% 1.2%
	1.28
Three Span	
CSB-A CSB-B	2.5%
CSB-C	1.8%
and the second second	1.0%
Four Span	
PCIB-A	1.7%



Table 2.6 Summary of the Frequency Analysis

	Fundamental Theor	Bending etical	Frequency (Hz)	ס מ מ ני	į.	. Additional
Bridge Type	Non Composite	Composite	Measured	Composite Action	Torsional Frequency(Hz)	Frequencies (Hz) Excited
Single Span	E					
SB-A-4	4.13	6.39	7.62	100%	8.40	9.96 14.4 22.5
SB-B-1 SB-C-1	4.45 3.62	6.95	6.77/7.07	1000	7.88	10.4 11.2 7.32 12.0
Two Span						
KCSB-A-1	2	2.71	2.73		3.32	4.39 4.69
KCSB-B-1	NA A	5.19	61.2		6 / . 7	4
KCSB-C-2	NA	3.73	3.81		4.10	9.08
KCSB-D-2	NA	2.62	2.83		3.03	3.41 4.54 6.30 8.89 15.8 20.7
KCSG-A-1	NA	2.27	2.34		2.83	80
KCSG-8-1	NA	2.20	2.22		4.49	8.30 8.89 15.8 20.7
KCPG-A-1	NA	2.36	2.44		3.12	4.10 7.81 8.11 17.6
KCPG-B-2	NA	2.29	2.25		2.83	3.61 3.91



Table 2.6 Continued

Action Composite Measured Action Composite Action A	Fundance	Fundamental Bending Frequency (Hz)	equency (Hz)		par in a d	
6.31 11.11 7.54 7% 4.96 8.13 5.27 1% 3.88 6.48 3.91 0% NA 15.43 13.5/16.7 NA 14.69 15.0/24.0 NA 13.66 13.41 NA 6.26 6.45	i	Theoretical Theoretical	Measured	Percent Composite Action	Fundamental Torsional Frequency(Hz)	Additional Frequencies (Hz) Excited
6.31 11.11 7.54 7% 4.96 8.13 5.27 1% 3.88 6.48 3.91 0% NA 15.43 13.5/16.7 NA 13.66 15.0/24.0 NA 13.66 13.41 NA 6.26 6.45	ree Span					
4.96 8.13 5.27 1% 3.88 6.48 3.91 0% NA 15.43 13.5/16.7 NA 14.69 15.0/24.0 NA 13.66 13.41 NA 6.26 6.45			7.54	7%	36.8	10.3 14.1 15.9
4.96 8.13 5.27 1% 3.88 6.48 3.91 0% 3.91 0% 15.43 13.5/16.7 NA 13.57 12.0/18.0 NA 13.66 13.41 NA 6.26 6.45			5.27	×-	5.96	7.42 8.20 9.7
NA 15.43 13.5/16.7 NA 14.69 15.0/24.0 NA 13.66 13.41 NA 6.26 6.45			3.91	% %	5.96 4.49	7.42 9.77 5.36
NA 13.66 15.0/24.0 NA 13.66 13.41 NA 6.26 6.45		15.43	13.6/16.7			11.0 thru 35.0 Hz 20.2 24.0 23.7
NA 13.66 13.41 NA 6.26 6.45 NA 6.26 6.45		14.69	15.0/24.0			
NA 6.26 6.45	B-B-1 NA	13.66	13.41		-	20.6 22.0 26.9
NA 6.26 6.45	ir Span					
NA A A A A A A A A A A A A A A A A A A		92-9	6.45		6.93	7.42 7.91 9.18
	B-A-2 NA	6.26	6.45		6.93	7.42 7.91 9.18



single span and two span bridges designed as composite structures were performing as anticipated; there was generally 100% composite action. The three span bridges designed as non-composite structures exhibited practically no composite action.

Also contained in Table 2.6 is a listing of the predominant frequencies excited for several typical bridges. It was considered beyond the scope of this investigation to seek to identify modes or energy sources corresponding to each of the frequencies excited. The importance of motion in the first torsional mode dictated that this frequency be experimentally identified; these values, along with the fundamental bending frequencies, are given in the table. The measured initial torsional mode frequency was found to be generally only slightly larger than the fundamental bending frequency. The complex nature of the actual forced bridge response due to a single vehicle traverse was especially evident from study of the frequency spectra generated.



CHAPTER 3

HUMAN SENSITIVITY TO VIBRATIONS

Human sensitivity to vibrations poses serious technical problems for engineers in various fields. In the field of transportation there is concern for comfort in automobiles, aircraft, and railway vehicles.

Structural engineers must consider wind-induced vibrations of tall buildings as well as floor vibrations in both buildings and bridges.

One of the goals of this research has been to identify what constitutes an objectionable level of vibration for pedestrians on bridges.

Human reactions to vibrations are both physiological and psychological. Low frequency, large amplitude vibrations, for example, are associated with sea sickness. On the other hand, when a person feels the traffic-induced vibration of a bridge, his reaction may be primarily psychological. He may associate this unexpected motion with poor design and possible collapse. Buildings and bridges are not supposed to move.

Some of the earliest measurements concerning human sensitivity to vibrations were carried out by Mallock in 1902 (23). Investigating complaints of unpleasant vibration caused by passing traffic to certain houses near Hyde Park, he found that amplitudes seldom exceeded .001 inch and that frequencies ranged from 10 to 15 Hz. From his results he deduced that it was acceleration which caused the discomfort and that an acceleration of one percent of gravity was noticeable.

Since Mallock's investigation, numerous experiments have been carried out in the field of human sensitivity to vibration. Wright and Green's report (24) contains an extensive bibliography. Experiments have been of two types: (1) people subjected to the vibration of actual structures in the field, and (2) people subjected to controlled



"shake table" vibrations in a laboratory. In these latter experiments tables are excited in simple harmonic motion of various amplitudes and frequencies. Results of these tests are usually presented in the form of "sensitivity curves", which delineate levels of vibration perception in the amplitude-frequency domain.

For harmonic motion, $y = a \sin 2$ nt, where a is the amplitude and n is the frequency in Hz, the maximum velocity, acceleration, and jerk are simply 2π an, $4\pi^2$ an 2π , and $8\pi^3$ an 3π , respectively. If human sensitivity is directly related to one of these three, then a curve corresponding to a particular level of sensitivity should appear as a straight line on a log-log plot. The slope of the line shows whether sensitivity depends on velocity, acceleration, or jerk.

Another early study was carried out by Reiher and Meister (25).

They subjected some ten people, aged 20 to 37 years, to vertical sinusoidal vibration without damping for about five minutes. Their results, shown in Figures 3.1 and 3.2, indicate that lower sensitivity levels depend on the product of maximum displacement and frequency, which is equivalent to velocity for harmonic motion. Higher sensitivity levels appear to be more nearly related to accelerations.

Lenzen (26) in a later study of the vibration of steel joist - concrete slab floors, suggested using the Reiher and Meister curves with the tolerance limits increased by a factor of ten if the amplitude decays to less than 10 percent of its initial magnitude in 5 to 12 cycles. Jacobsen and Ayre (27) showed that 3 percent critical damping is required to meet Lenzen's requirement. Kropp (5) found damping of up to 2.5 percent for highway bridges.



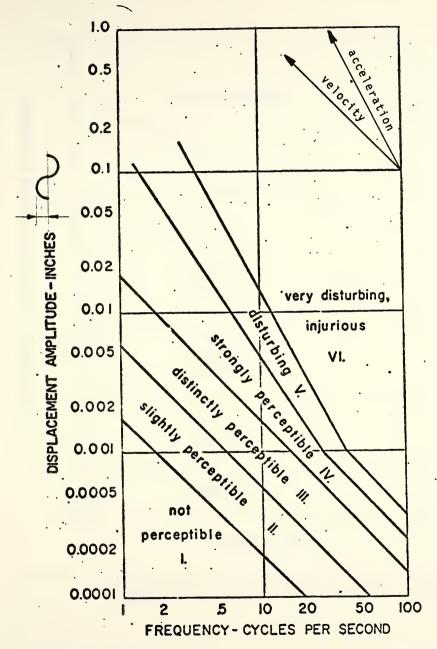


Figure 3.1. Domains of Various Strengths of Sensations for Standing Persons Subject to Vertical Vibration, After Reiher and Meister.



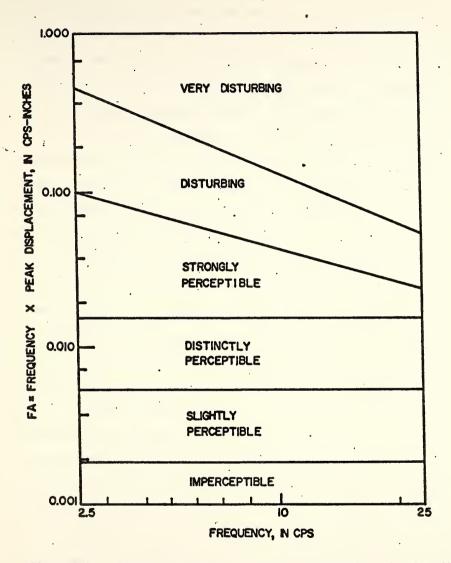


Figure 3.2 Domains of Various Strength of Sensations for Standing Person Subject to Vertical Vibration, After Reiher and Meister.



More recently, Wiss and Parmelee (28) investigated the effect of damping upon human sensitivity to vibration. Standing subjects were asked to rate various combinations of frequency, peak amplitude, and damping on a perception scale of 1 to 5. Frequencies ranged from 2.5 to 25 Hz, peak displacements from .0001 to .1 inch, and damping from .01 to .16 of critical damping. Like Reiher and Meister, they found sensitivity to be proportional to the product of maximum displacement and frequency. For a given sensitivity rating they determined that the amplitude-frequency product could be approximately twice as much when the damping was increased from zero to 3 percent of critical.

Although the modified Reiher-Meister curves are fairly well accepted for building design, many other comfort limits have been proposed, especially in the transportation field. Figure 3.3 shows the results of 15 different investigators. As can be seen, there is not complete agreement among the results; however these curves do seem to indicate that comfort is related to acceleration in the 5 to 10 Hz frequency range.

For vertical vibration limits for automobile passenger comfort, Janeway (29) has proposed the curve shown in Figure 3.4. For 1-6 Hz, he indicates that sensitivity is related to jerk, for 6-20 Hz acceleration, and for 20-60 Hz velocity. Note that his results are similar to most of those shown in Figure 3.3.

Experimental data from several investigators has been summarized by Goldman (30) as shown in Figure 3.5. The curves are plotted through mean values with the vertical bars indicating standard deviations.

Wright and Green have expanded Goldman's results to give more levels of sensitivity as shown in Figure 3.6. In an effort to select a single quantitative measure of vibration sensitivity for design, Wright and Walker (31) have decided that in the frequency range of interest for



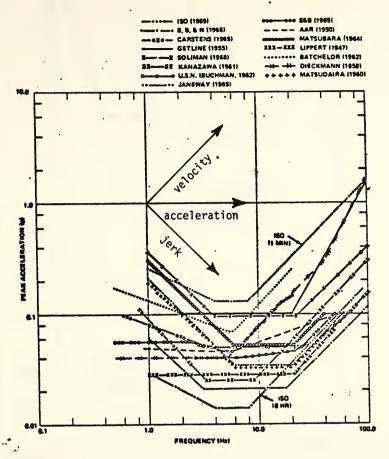
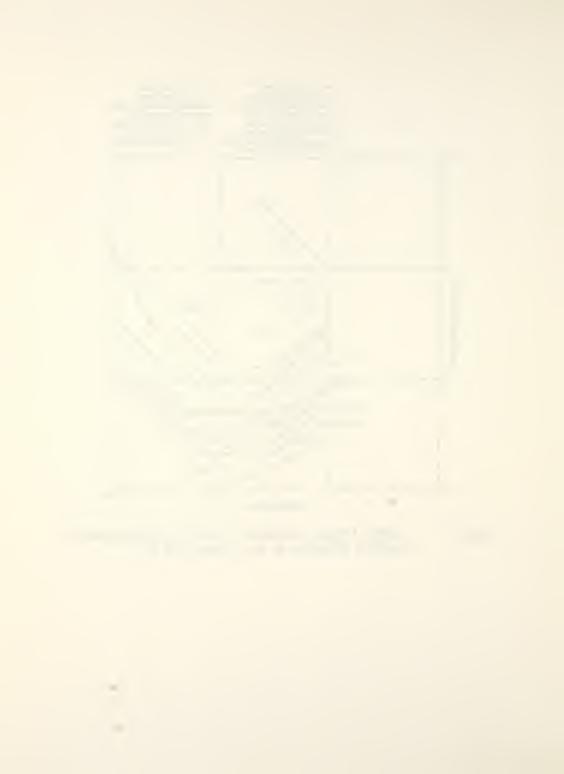


Figure 3.3 Comfort Limits Recommended by Various Investigators for Vertical Vibration or Axis Unspecified [32].



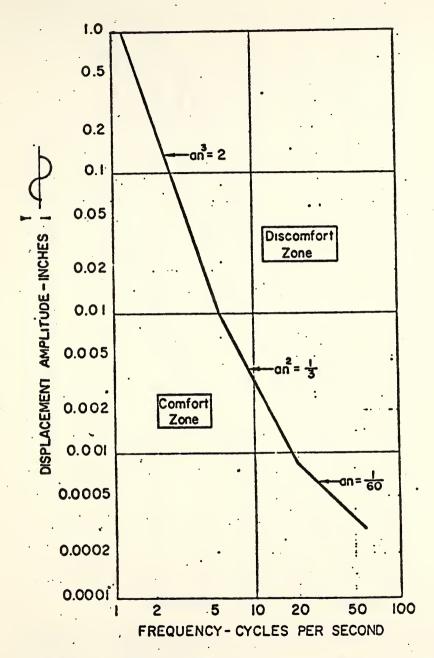


Figure 3.4 Vertical Vibration Limits for Automobile Passenger Comfort After Janeway.



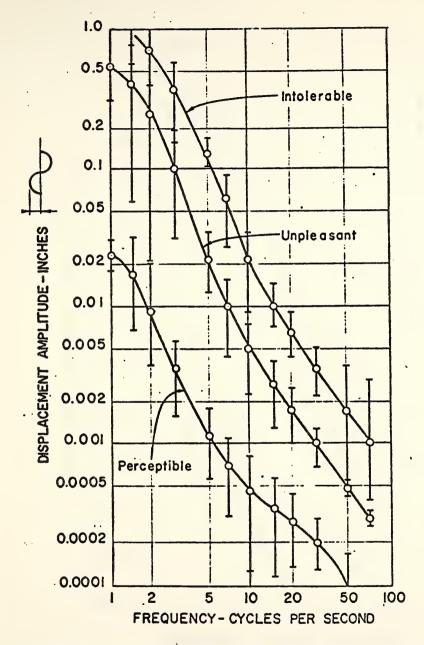
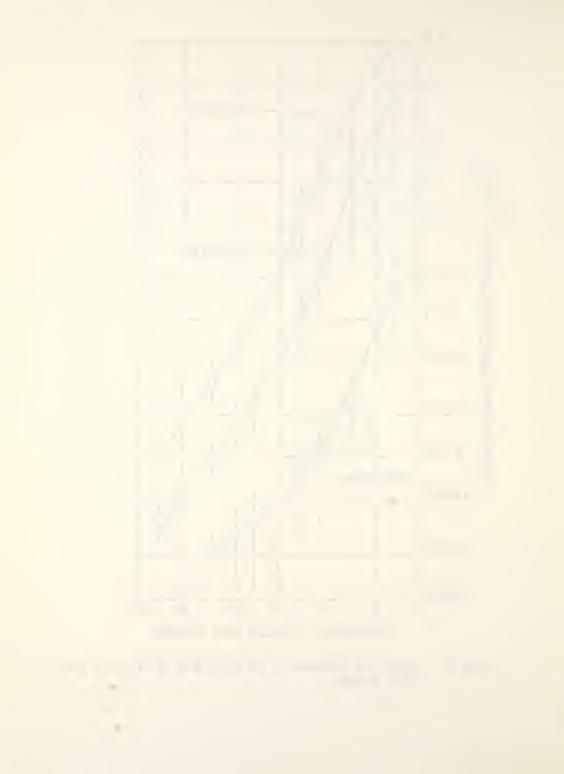


Figure 3.5 Subjective Responses of the Human Body to Vibratory Motion, After Goldman



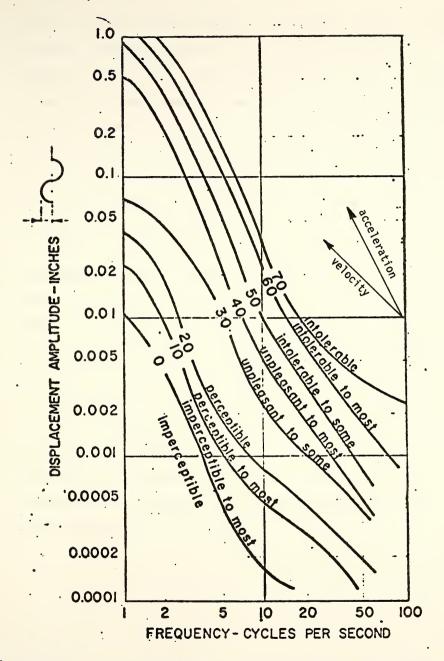


Figure 3.6 Contours of Equal Sensitivity to Vibration, After Goldman



bridges (1 to 20 Hz) Goldman's curves are essentially constant acceleration lines. Goldman's "unpleasant to some" curve corresponds to an acceleration of 10 in/sec². Because these sensitivity curves are based on steady state harmonic motion of many cycles of duration, Wright and Walker have proposed a ten-fold increase (to 100 in/sec²) in the magnitude of the maximum acceleration for the folerance level for transient bridge vibrations. This magnitude of acceleration is definitely perceptible but is well within the tolerable range if pedestrains are informed that some motion is to be expected.

Shahabadi's review of the literature has shown that there is no simple single parameter which can completely represent the shadings of human sensitivity to vibration. Within the relevant frequency range for highway bridges, strong arguments have been made for using either a velocity limit or an acceleration limit. Fortunately, it appears that either velocity or acceleration could be used as a serviceability criterion for bridge design. The focus of this research project has been on accelerations. The 100 in/sec² acceleration limit proposed by Wright and Walker appears to be reasonable although somewhat higher than suggested by others, probably because of the somewhat arbitrary tenfold increase in magnitude taken to account for the shorter duration of the large amplitude vibrations.



CHAPTER 4

COMPARISON OF RESULTS

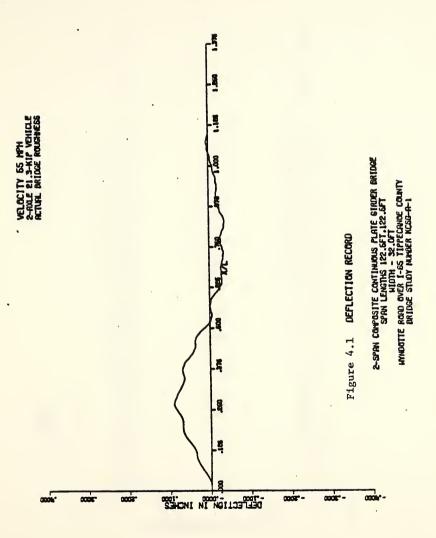
4.1 ROADWAY ROUGENESS

In his analytical studies, Aramraks (2) found roadway roughness to be one of the most significant factors influencing the vibration of highway bridges. By adjusting the length of assumed sinusoidal surface irregularities, he obtained maximum accelerations as much as 10 to 20 times as large as for a smooth roadway. Although convenient for calculation purposes, the sinusoidal variation is not necessarily realistic.

When Shahabadi (6) first attempted to calculate the maximum response of some of the test bridges he assumed a smooth roadway. The accelerations predicted by the computer program were significantly smaller than those measured in the field. To obtain a reasonable representation of actual roadway roughness, field crews carefully measured profiles for three of the test bridges, which were then used as input data to improve the analytical predictions.

From the measured profiles, a method was developed to simulate a typical surface roughness for input data for the computer program. The roughness simulation takes the form of a cosine series with random phase angles. The relationship between the amplitudes and frequencies of the cosine terms is given by a Weibull distribution function. A reasonable comparison of computed responses was obtained for the control vehicle crossing a two span test bridge with the actual and a simulated profile. The results are shown in Figures 4.1 - 4.4. Several different simulated profiles produced dynamic responses in the same order of magnitude for this bridge. Using this type of simulated roadway roughness, the resonance phenomena observed by Aramraks, using sinusoidal roughness, did not occur.







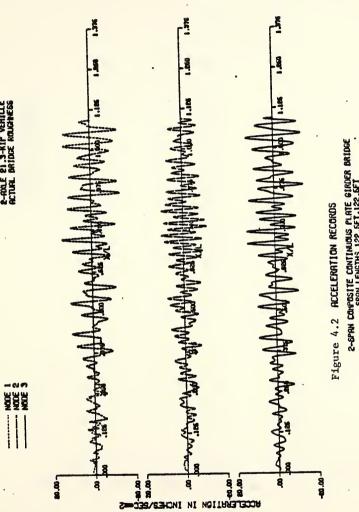
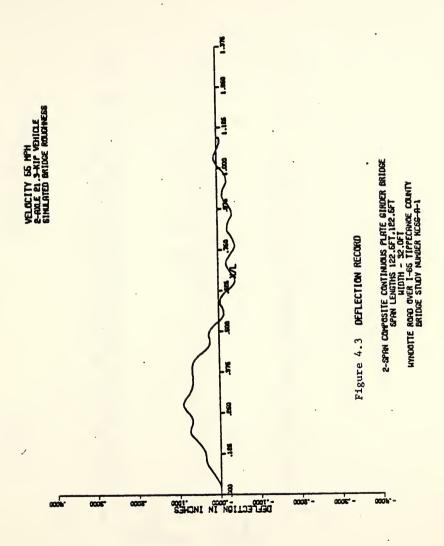


Figure 4.2 ACCELERATION RECORDS

2-6PAN COMPOSITE CONTINUOUS PLATE GIRDER BRIDGE
SPAN LENGTHS 122.5FT.122.5FT
WINDOTTE ROOD OVER 1-65 TIPPECANGE COUNTY
BRIDGE STUDY NUMBER KCSG-R-1





)



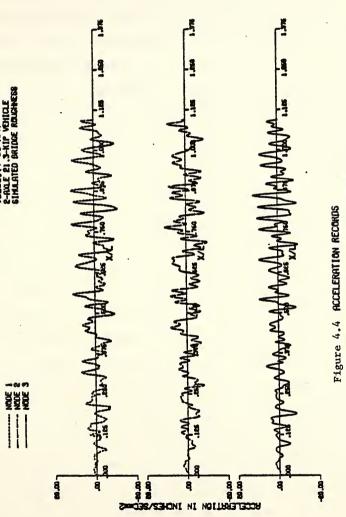


Figure 4.4 ACCLERATION RECORDS
2-SPAN CAPPOSITE CONTINUOUS PLATE GIRDER BRIDGE
SPAN LENGTHS 122.5FT.122.6FT
HIDTH 32.0FT
WYNDGTTE ROAD OVER 1-65 INFFERNEE COUNTY
BRIDGE STUDY NUMBER KCSG-0-1



4.2 COMPARISON OF ACTUAL AND COMPUTED DYNAMIC RESPONSES

Some of limitations of the computer programs became evident as attempts were made to compare results. Although the single span program does consider the bridge to be a flat plate supported on equally spaced beams, the vehicle is modeled as a single axle sprung mass. There is no damping in either the bridge or the vehicle, and only sinusoidal roadway roughness can be considered.

The bridge is modeled as a single continuous beam with lumped masses for the multispan analysis program. The vehicle model is much more sophisticated than for the simple span program - considering multiple axles, axle spacing, tire and spring stiffness, and the damping effect of shock absorbers. This program can also consider an arbitrary surface roughness. The only disadvantage of the multispan program is that it cannot consider rolling of the vehicle or torsion of the bridge because the bridge is modeled as a single beam.

Using the multispan program, Aramraks found that the midspan accelerations for a two span bridge were nearly the same for two and three axle vehicles but increased significantly for a single axle vehicle of the same weight. This indicates that the simple span program, which allows only a single axle vehicle, would overestimate the midspan accelerations. On the other hand, by assuming a smooth roadway the simple span program underestimates the accelerations. The net result is that some reasonable estimates of the actual accelerations were obtained. The simple span analysis did correctly predict the increase in measured accelerations with the path of the vehicle moved transversely closer to the curb. Some results are shown in Table 4.1.



Using actual or simulated roadway roughness, the two and three span programs predicted accelerations which were of the same order of magnitude as the measured values. Higher accelerations were measured in the field when the vehicle was traveling close to the curb. These higher accelerations are due to torsional modes of vibration which are not considered in analysis. Some results are shown in Tables 4.2 and 4.3.



TABLE 4.1. COMPARISON OF MEASURED AND CALCULATED PEAK DYNAMIC RESPONSES FOR A 72' SIMPLE SPAN STEEL BEAM BRIDGE (SB-C-1)

	Measured		Calculated	
Vehicle Position	Deflection (in)	Acceleration (in/sec ²)	Deflection (in)	Acceleration (in/sec ²)
Centerline	.005	11.2	.003	12.5
Travel Lane	.025	16.8	.023	14.1
Curb Lane	.044	18.6	.054	15.2

TABLE 4.2. COMPARISON OF MEASURED AND CALCULATED PEAK DYNAMIC RESPONSES FOR A TWO SPAN CONTINUOUS COMPOSITE PLATE GIRDER BRIDGE (KCSG-A-1)

	Measured		Calculated	
Vehicle Position	Deflection (in)	Acceleration (in/sec ²)	Deflection (in)	Acceleration (in/sec ²)
Centerline	.09	13.9	.09	11.4
Travel Lane	.12	14.2		
Curb Lane	.15	15.7		



TABLE 4.3. COMPARISON OF MEASURED AND COMPUTED PEAK
DYNAMIC RESPONSES FOR A THREE SPAN CONTINUOUS
NON-COMPOSITE STEEL BEAM BRIDGE (CSB-C-1)

	Measured		Calculated	
Vehicle Position	Deflection (in)	Acceleration (in/sec ²)	Deflection (in)	Acceleration (in/sec ²)
Centerline	.022	12.3	.027	12.7
Travel Lane	.045	15.1		
Curb Lane	.065	16.8		



CHAPTER 5

DESIGN IMPLICATIONS AND CONCLUSIONS

5.1 IMPLICATIONS FOR DESIGN

In the time which has passed since the start of this research program, there have been two significant changes in North American bridge codes. The first concerns a relaxation of the deflection and depthspan ratio limitations in the 1977 AASHTO Bridge Specifications. In particular, Section 1.7.6 (Deflections) states, "The foregoing [live load deflection] requirements as they relate to beam or girder bridges may be exceeded at the discretion of the designer. For considerations to be taken into account when exceeding these limitations, reference is made to 'Bulletin No. 19, Criteria For the Deflection of Steel Bridges,' available from AISI."

In this report, Wright and Walker recommend that annoying vibrations be controlled by limiting the dynamic component of bridge acceleration to 100 in/sec², which is based upon Goldman's sensitivity curves with a tenfold increase in magnitude for the short duration of the maximum acceleration. They propose that the acceleration be calculated simply as the product of the dynamic displacement $\delta_{\rm D}$ and the square of the fundamental (circular) frequency.

$$a = \delta_D (2\pi f)^2 \le 100 \text{ in/sec}^2$$

The dynamic displacement is determined from the maximum static deflection δ_S , which is calculated conventionally for a wheel load distribution factor of 0.7. The dynamic displacement is typically about 30% of the maximum static deflection.

$$\delta_{\rm D} = (.15 + \frac{\rm v}{2\rm fL}) \delta_{\rm S}$$



where v = vehicle speed

f = fundamental frequency of bridge

L = span length

Another somewhat similar approach to the control of vibrations is contained in the new Ontario Highway Bridge Design Code (33). The previous AASHTO-type deflection controls for steel bridges have been completely deleted. The deflection limit is now given as a function of the fundamental bending frequency in an attempt to correlate bridge response with human sensitivity. Three vibration levels, depending on the degree of usage by pedestrians are specified in curve form as shown in Figure 5.1. The average slopes of these curves correspond approximately to constant velocity lines for harmonic motion.

The Wright and Walker serviceability limit with $\delta_{\rm D}$ = .3 $\delta_{\rm S}$ is also plotted in Figure 5.1 for comparative purposes. Although the acceleration line has a much different slope than the curves, the two approaches give similar deflection limits in the middle range of bridge fundamental frequencies.

Certainly both the Wright and Walker proposal and the Ontario Highway Bridge Design Code present criteria for vibration control which can be easily used by designers. Both require calculation of only the static deflection and the fundamental frequency of the bridge - quantities which can be readily and reliably determined.

Wright and Walker's method was used to estimate the maximum acceleration for nine of the test bridges. Static deflections were calculated for a 0.7 wheel load distribution factor, assuming a HS20 truck traveling at a speed of 55 mph. Although the three simple span bridges were designed and built as non-composite structures, properties of the composite section were used to calculate static deflections and natural



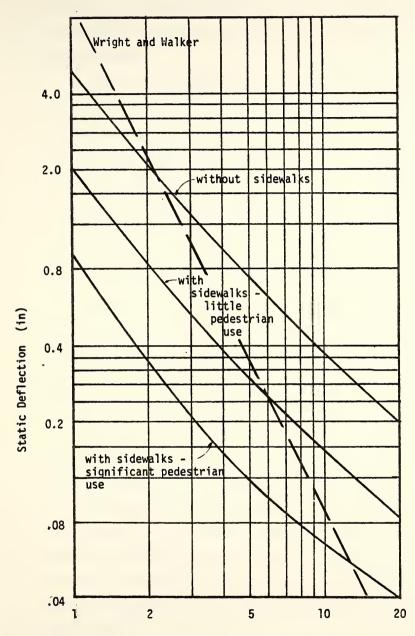


Figure 5.1 First Flexural Frequency (Hz)

Ontario Bridge Code - Vibration Control



frequencies. Frequencies computed in this way compared favorably with those measured in the field. All of the two span bridges were composite construction.

A summary of the bridge properties and estimated accelerations are shown in Table 5.1. Also listed in this table are the peak accelerations measured in the field under actual traffic. The estimated values do, in general, indicate correct trends. The accelerations of the simple span bridges were properly predicted to be larger than those of the two span bridges, primarily because of the higher natural frequencies. Because several large assumptions were necessary to develop this simple estimation formula, a high degree of precision is not to be expected. For example, a bridge is assumed to vibrate in its fundamental bending mode, while Kropp found two or more dominant frequencies in the free vibration of many of the test bridges. Also, the dynamic displacement is calculated only for an "average" roadway roughness. Finally, the use of the 0.7 wheel load distribution factor for all beam spacings seems questionable.

Experimentally determined fundamental bending and torsional frequencies as well as peak displacements and accelerations for 14 series of steel beam and girder test bridges are shown in Table 5.2. For each bridge Shahabadi computed the product of the peak displacement and the square of the fundamental circular frequency. Results are compared with measured peak accelerations in Figure 5.2. Although it is not as theoretically sound to use total displacements instead of dynamic displacements, the acceleration predictions are surprisingly accurate.



Comparison of Measured Peak Accelerations With Those Predicted by Wright and Walker's Equation Table 5.1

lerations	Measured in/sec ²	68	27	20	ć	33	133	42	ဓ	23	27
Peak Acce	Predicted* Measure, in/sec ² in/sec ⁴	116	128	118	5	09	26	24	27	56	28
	Fundamental Frequency (Hz)	6.39	6.95	4.82	,	2.19	3.73	2.62	2.27	2.20	2.36
	Static* Defl (in)	0.284	0.263	0.485	4	0.965	0.351	0.285	0.452	0.294	0.448
	EI Beam (K-in ²)	.604 × 109	.490 × 109	.636 × 109		×	×	1.898 x 109	×	×	×
	Span (ft)	09	22	75	6	103.5	9/	96	122.5	128	129
	Bridge Identification	Single Span SB-A-1	SB-B-1	SB-C-1	Two Span	KC38-8-1	KCSB-C-2	KCSB-D-2	KCSG-A-1	KCSG-B-1	KCPG-A-1

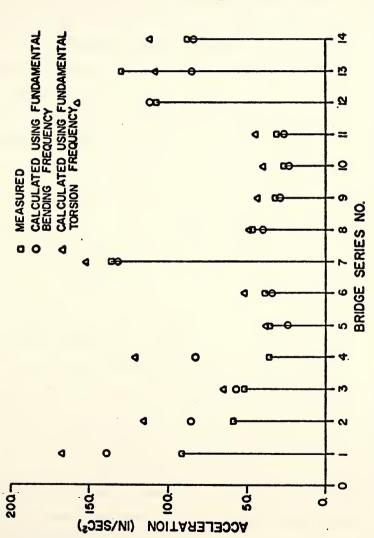
*Based on HS20 Truck Traveling 55 MPH.



Measured Fundamental Frequencies and Maximum Measured Accelerations and Deflections for the Bridges in the Study Table 5.2

	Fundamental Fre	Frequency (HZ)		
		Torsion	Deflection (IN)	Acceleration (IN/SEC**2)
Single Span SB-A-1 thru SB-A-5 7.62 SB-B-1 thru SB-B-3 6.77 SB-C-1 4.88	62 77 88	8.40 7.88 5.27	.060	89. 57. 50.
Two Span KCSB-A-1 and KCSB-A-2 2.73 KCSB-B-1 and KCPG-B-2 2.25 KCSB-C-1 thru KCSB-C-4 3.81 KCSB-D-1 and KCSB-D-2 2.83 KCSG-A-1 and KCSG-A-2 2.34 KCSG-B-1 and KCSG-A-2 2.34	73 25 81 83 44	3.32 2.73 4.10 3.03 3.12	.277 .124 .154 .228 .123 .131	34. 33. 133. 42. 23.
Three Span CSB-A-1 thru CSB-A-4 7.5 CSB-B-1 thru CSB-B-4 5.2 CSB-C-1 3.9	7.54 5.27 3.91	8.96 6.96 8.96	.049 .075 .137	105. 124. 83.





Comparison of the Maximum Measured Accelerations with the Accelerations Obtained from the Simplified Method

Figure 5.2



5.2 CONCLUSIONS AND RECOMMENDATIONS

- 1) Analytical studies have shown roadway roughness to be a significant factor influencing bridge deck accelerations. Rougher decks do cause higher accelerations. Although a roughness index might be incorporated into a simple acceleration estimation formula, the condition of the deck surface varies with time. The best recommentation for vibration control is to build and maintain smooth roadways and smooth transitions form the approach slab to the bridge deck.
- 2) Moderate success has been achieved in checking field measurements analytically. Without precise knowledge of the initial conditions of the vehicle and the roadway roughness, a computer program based upon a perfect model of the system cannot yield a precise dynamic response history for a vehicle crossing. Also the models underlying the computer programs used in this study lacked some of the sophistication necessary for more accurate results. For example, in modeling a two span bridge as a single continuous beam, rolling of the vehicle and torisional response of the bridge are lost. The computer programs did, of course, properly establish trends, identify significant parameters, and predict peak responses.
- 3) The human body is sensitive to motion. Both velocity and acceleration criteria recently have been proposed and could be used successfully to limit bridge vibrations to levels which are not objectionable to pedestrians maintenance workers, cyclists, etc. Endorsement of the Wright and Walker recommendations for vibration control by AASHTO is a needed improvement in bridge design practice. Considering the complexities of computing the actual dynamic response history of a bridge, the simple displacement times frequency squared



- expression yields a reasonable and practical estimate of peak acceleration.
- simple span and continuous beam bridges and to reduce the data to obtain vibrational characteristics. Good comparison were obtained between twice-differentiated displacements and measured accelerations and between twice-integrated accelerations and measured displacements. The relatively low levels of accelerations measured for steel and concrete beam bridges seem to indicate that more flexible designs would still give vibration levels which ere not objectionable. The next logical step in this research would be to design and build a bridge more flexible than permitted by previous AASHTO rules using the Wright and Walker guidelines for vibration control and to monitor its dynamic performance under actual traffic and an instrumented control vehicle.



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